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Glover et al.

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[45] Date of Patent: Jun. 13, 1989

[54] FAST REMAINDER DECODING FOR A REED-SOLOMON CODE

[75] Inventors: **Neal Glover**, Broomfield; **Trent Dudley**, Littleton, both of Colo.

[73] Assignee: **Data Systems Technology Corp.**, Broomfield, Colo.

[21] Appl. No.: **12,824**

[22] Filed: **Feb. 10, 1987**

[51] **Int. Cl.**⁴ **G06F 11/10**
[52] **U.S. Cl.** **371/37**; 371/39
[58] **Field of Search** 371/37, 38, 39, 40

[56] **References Cited**

U.S. PATENT DOCUMENTS

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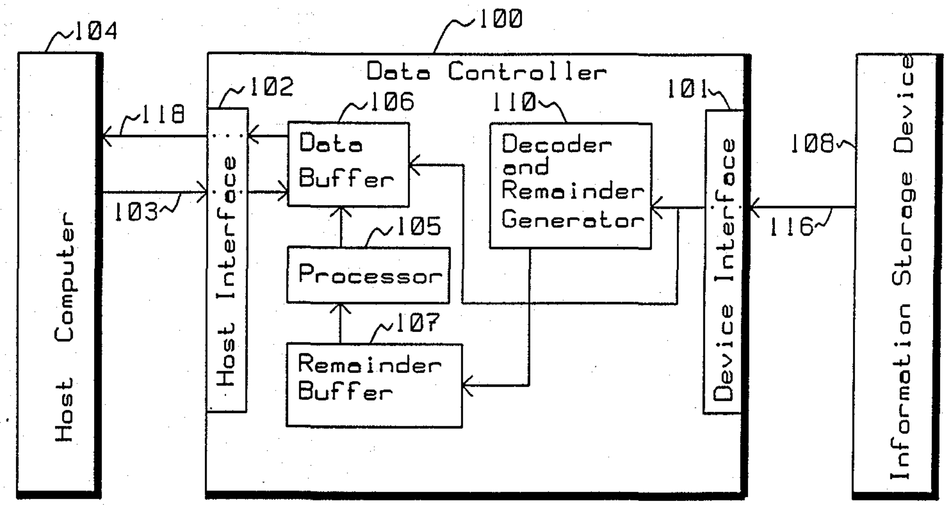
Primary Examiner—Michael R. Fleming
Attorney, Agent, or Firm—James R. Young

[57] **ABSTRACT**

Apparatus and methods are disclosed for providing fast decoding of Reed-Solomon and related codes. Cases of one and two data symbol errors are decoded directly from the remainder using a large pre-computed table without calculating syndromes. Techniques for decoding cases of more than two errors are given where an optimized Chien search is used when more than four errors remain; when four or fewer errors remain, the Chien search is eliminated in favor of locating an error by direct solution of the error locator polynomial. The error locator and syndrome polynomials are adjusted after each error is found, and the error evaluator polynomial need not be computed.

47 Claims, 21 Drawing Sheets



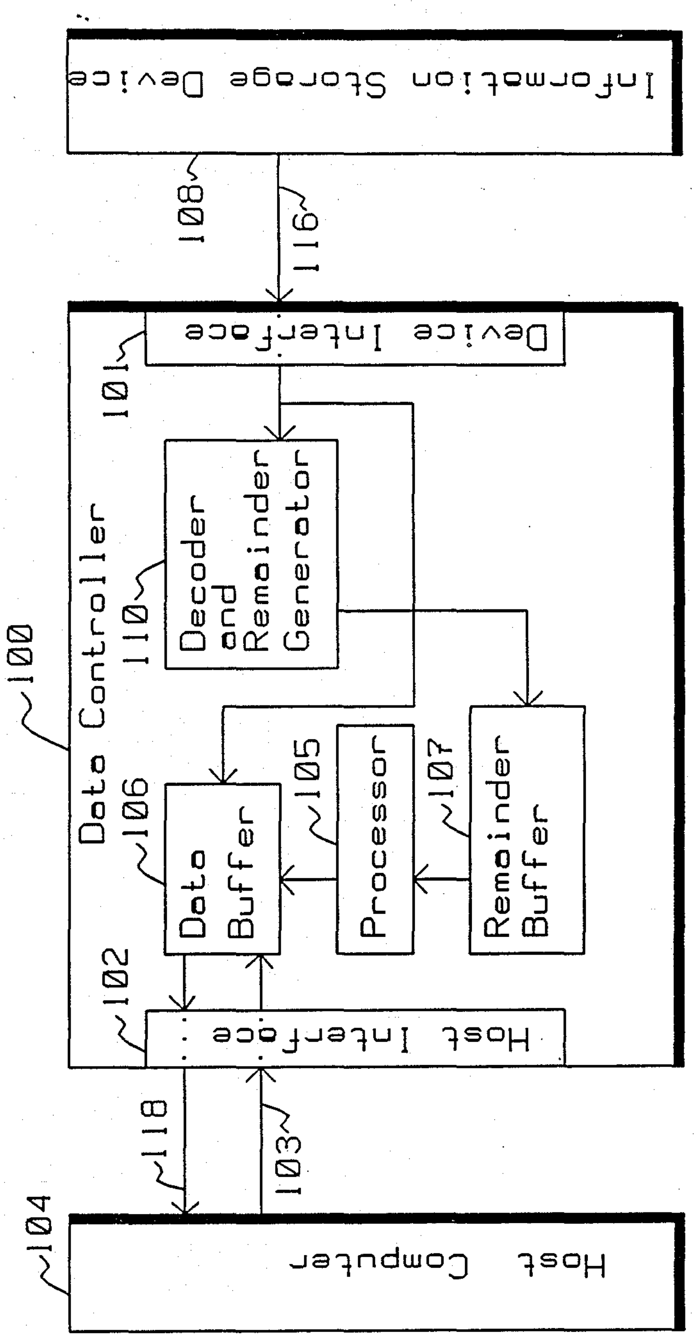

Information Storage Device
108
116
Device Interface
101
100
Data Controller
110
Decoder and Remainder Generator
106
Data Buffer
105
Processor
107
Remainder Buffer
102
Host Interface
118
103
104
Host Computer

FIG. 1

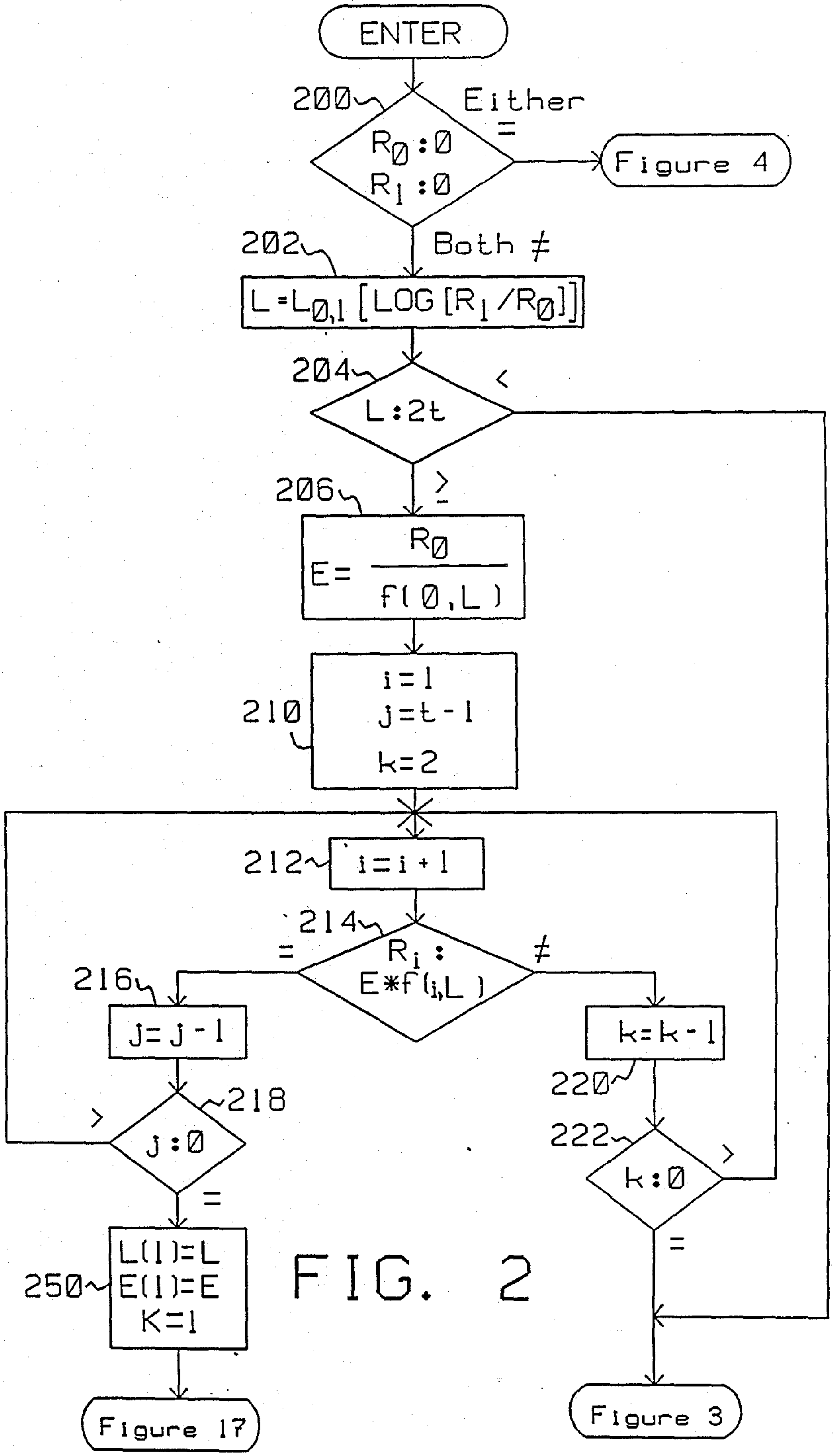

ENTER
200
$R_0 : 0$
$R_1 : 0$
Either =
Figure 4
Both ≠
202
$L = L_{0,1} [LOG [R_1 / R_0]]$
204
L:2t
<
≥
206
$E = \frac{R_0}{f(0,L)}$
210
i=1
j=t-1
k=2
212
i=i+1
214
$R_i :$
$E * f(i,L)$
=
≠
216
j=j-1
218
j:0
>
=
250
L(1)=L
E(1)=E
K=1
Figure 17
k=k-1
220
222
k:0
>
=
Figure 3

FIG. 2

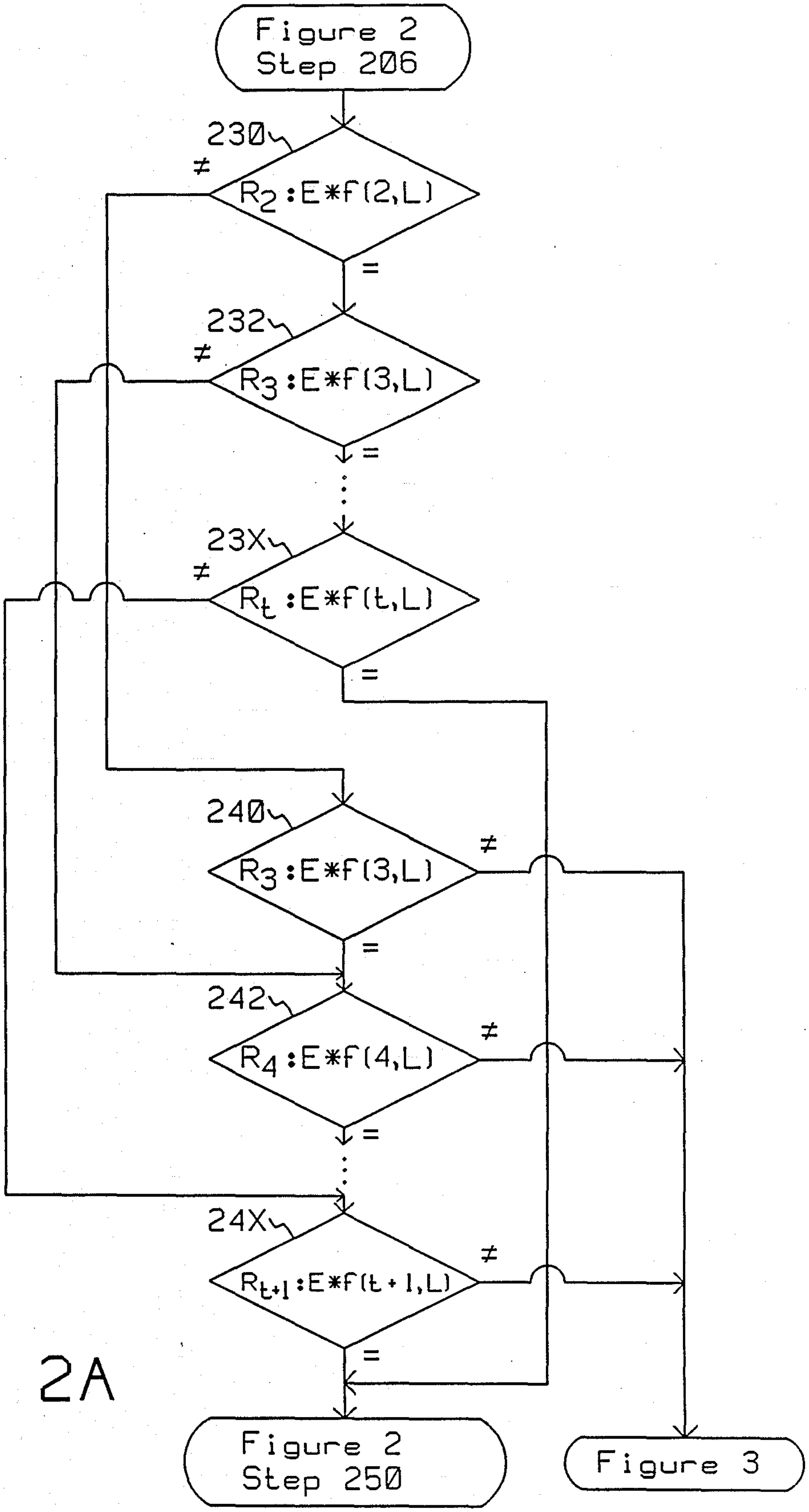


Figure 2
Step 206
230
≠
R_2 : E*f(2,L)
=
232
≠
R_3 : E*f(3,L)
=
23X
≠
R_t : E*f(t,L)
=
240
R_3 : E*f(3,L)
≠
=
242
R_4 : E*f(4,L)
≠
=
24X
R_{t+1} : E*f(t+1,L)
≠
=
Figure 2
Step 250
Figure 3

FIG. 2A

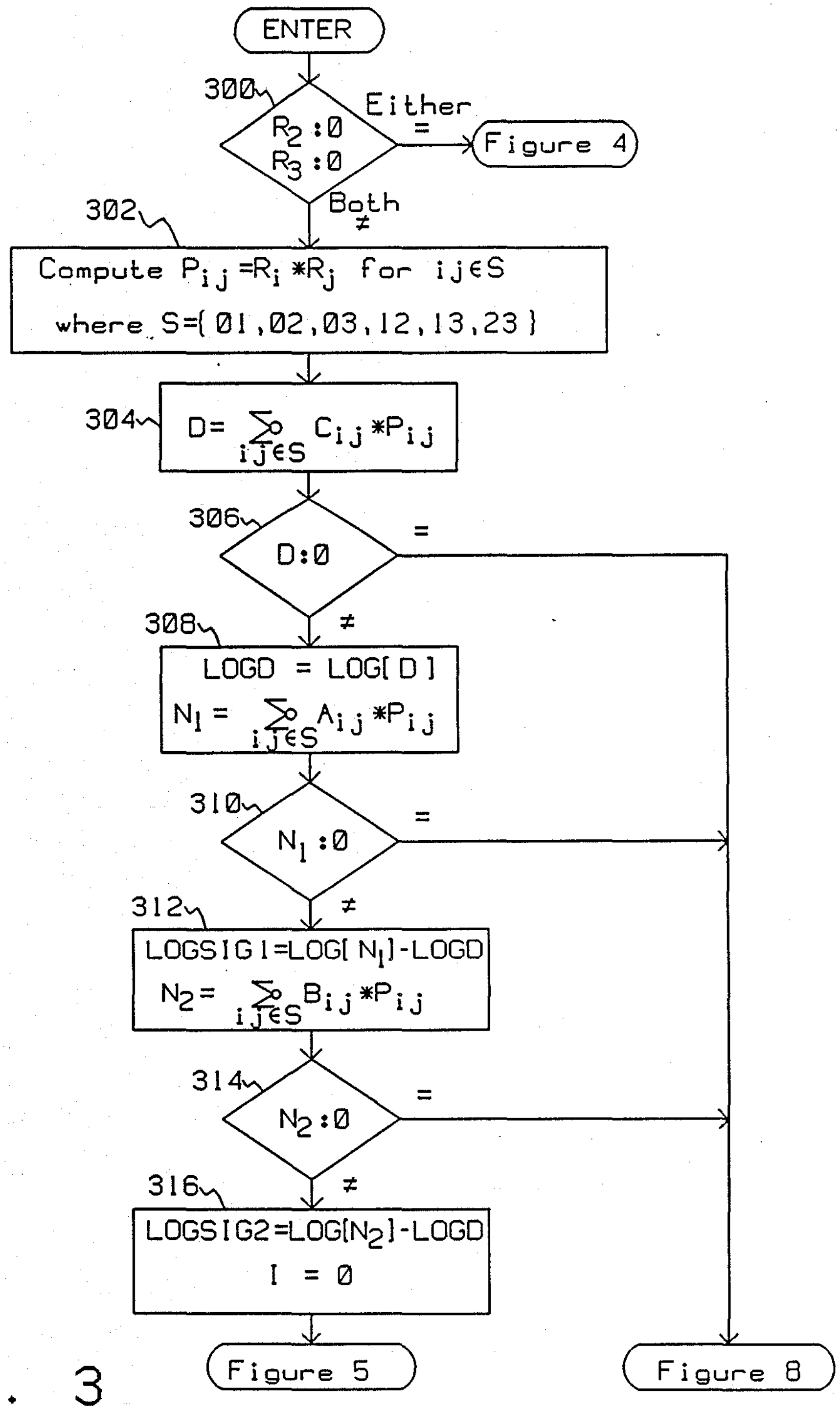

ENTER
300
$R_2:0$
$R_3:0$
Either =
Figure 4
Both ≠
302
Compute $P_{ij}=R_i*R_j$ for $ij\in S$
where $S=\{01,02,03,12,13,23\}$
304
$D=\sum_{ij\in S} C_{ij}*P_{ij}$
306
D:0
=
≠
308
LOGD = LOG[D]
$N_1=\sum_{ij\in S} A_{ij}*P_{ij}$
310
$N_1:0$
=
≠
312
LOGSIG1=LOG[N_1]-LOGD
$N_2=\sum_{ij\in S} B_{ij}*P_{ij}$
314
$N_2:0$
=
≠
316
LOGSIG2=LOG[N_2]-LOGD
I = 0
Figure 5
Figure 8

FIG. 3

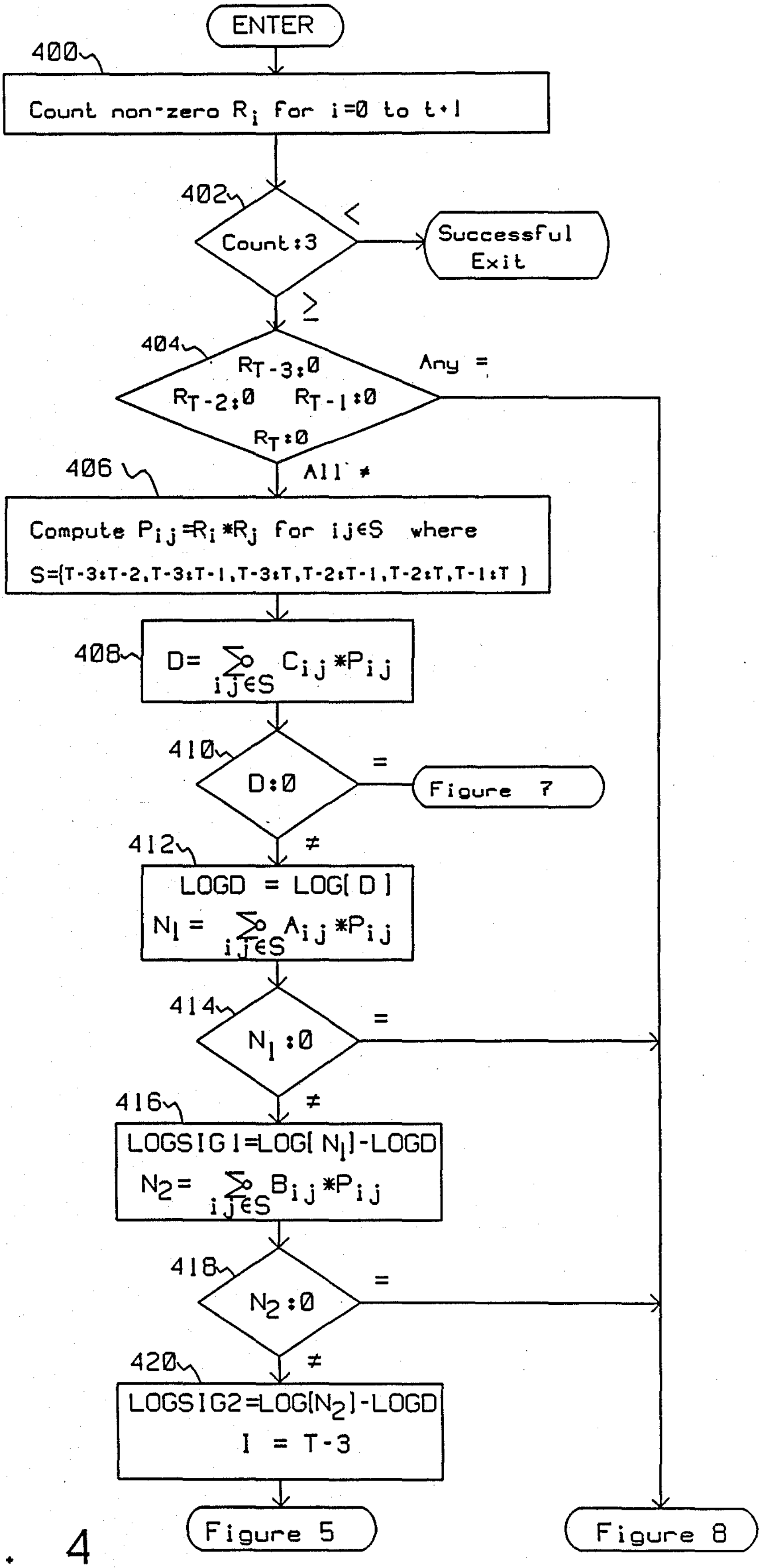

ENTER
400
Count non-zero R_i for i=0 to t+1
402
Count:3
<
Successful Exit
≥
404
R_{T-3}:0
R_{T-2}:0
R_{T-1}:0
R_T:0
Any =
All ≠
406
Compute $P_{ij}=R_i*R_j$ for ij∈S where
S={T-3:T-2, T-3:T-1, T-3:T, T-2:T-1, T-2:T, T-1:T }
408
$D=\sum_{ij\in S} C_{ij}*P_{ij}$
410
D:0
=
Figure 7
≠
412
LOGD = LOG[D]
$N_1=\sum_{ij\in S} A_{ij}*P_{ij}$
414
N_1:0
=
≠
416
LOGSIG1=LOG[N_1]-LOGD
$N_2=\sum_{ij\in S} B_{ij}*P_{ij}$
418
N_2:0
=
≠
420
LOGSIG2=LOG[N_2]-LOGD
I = T-3
Figure 5
Figure 8

FIG. 4

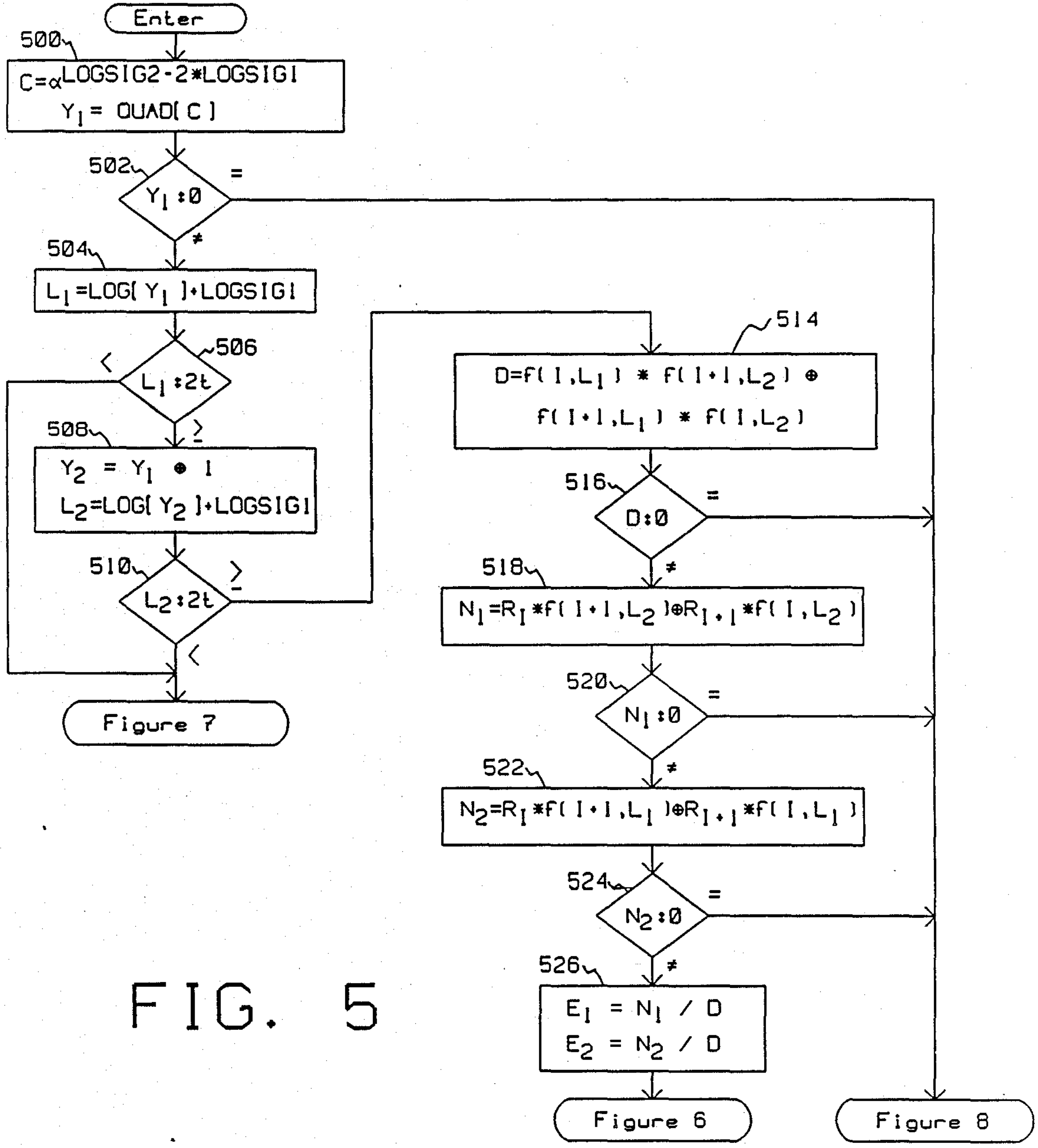

Enter
500
C=αLOGSIG2-2*LOGSIG1
Y1 = QUAD[C]
502
Y1 :0
=
≠
504
L1 =LOG[Y1]+LOGSIG1
506
L1 :2t
<
≥
508
Y2 = Y1 ⊕ 1
L2=LOG[Y2]+LOGSIG1
510
L2 :2t
≥
<
Figure 7
514
D=f(I,L1) * f(I+1,L2) ⊕
f(I+1,L1) * f(I,L2)
516
D:0
=
≠
518
N1 =RI *f(I+1,L2)⊕RI+1 *f(I,L2)
520
N1 :0
=
≠
522
N2=RI *f(I+1,L1)⊕RI+1 *f(I,L1)
524
N2 :0
=
≠
526
E1 = N1 / D
E2 = N2 / D
Figure 6
Figure 8

FIG. 5

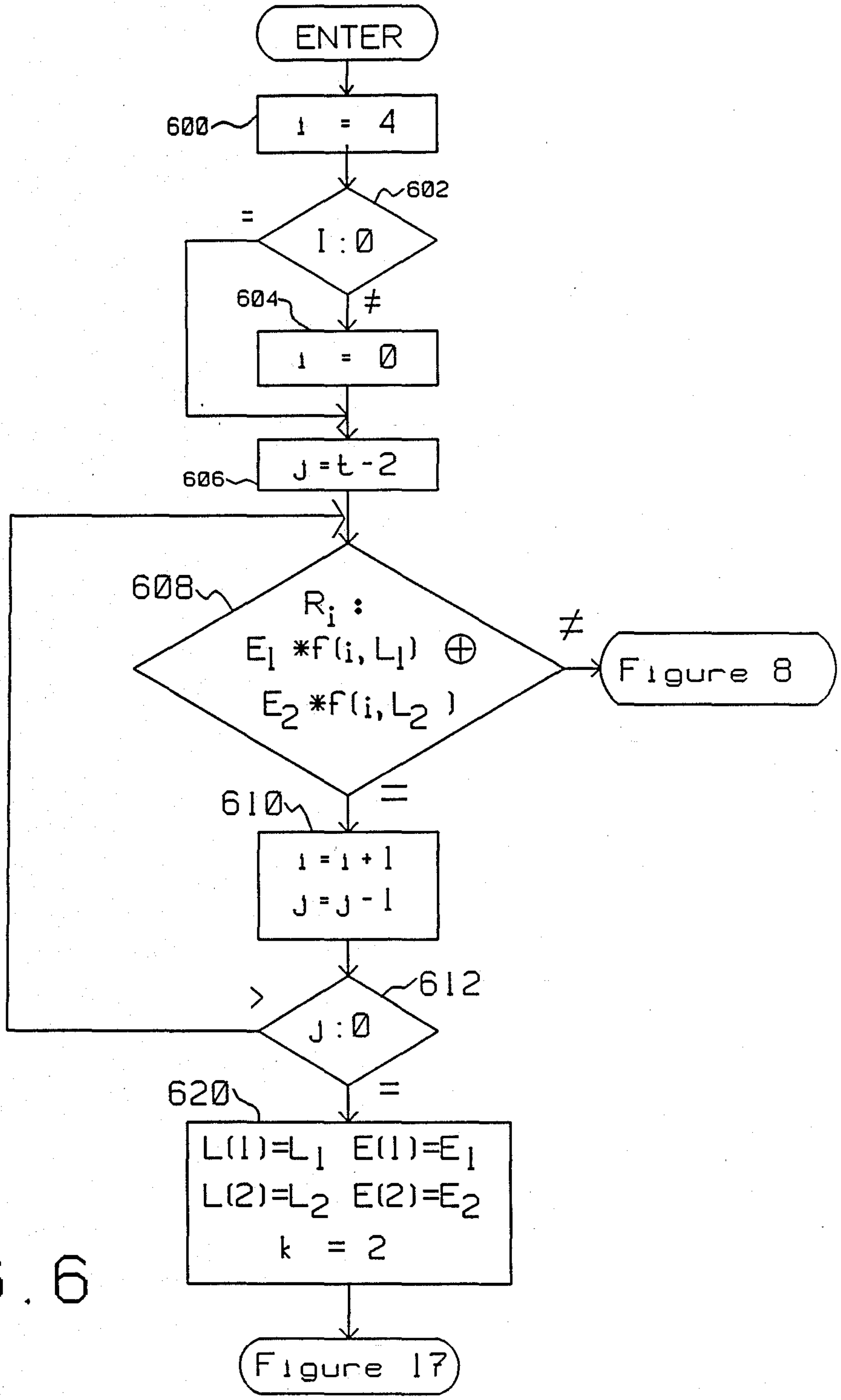

ENTER
600
i = 4
602
I : 0
=
≠
604
i = 0
j = t - 2
606
608
R_i :
$E_1 * f(i, L_1) \oplus$
$E_2 * f(i, L_2)$
≠
Figure 8
=
610
i = i + 1
j = j - 1
612
>
j : 0
=
620
$L(1)=L_1$ $E(1)=E_1$
$L(2)=L_2$ $E(2)=E_2$
k = 2
Figure 17

FIG. 6

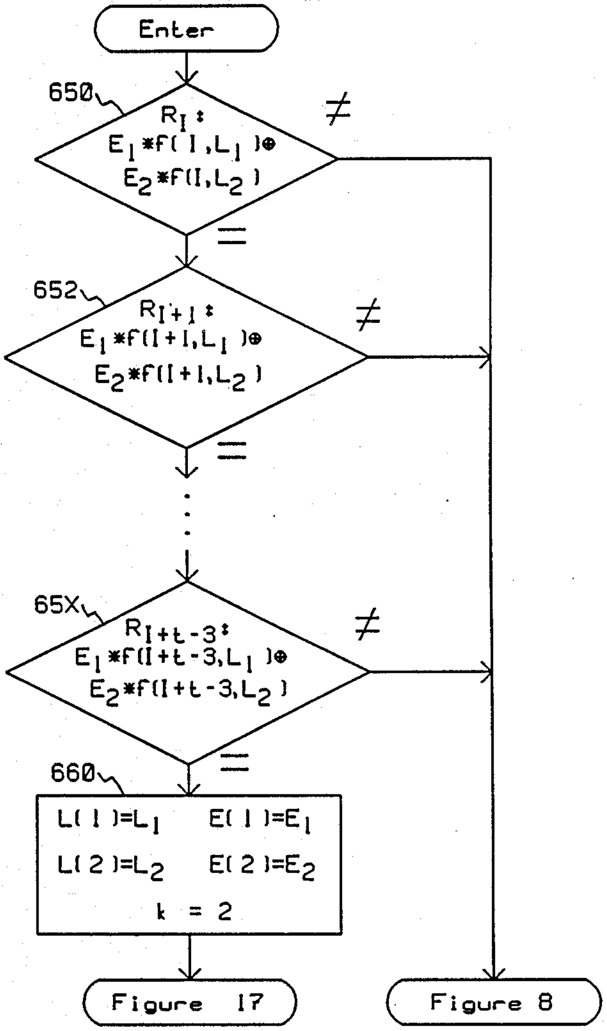

Enter
650
R_I:
$E_1*f(I,L_1)\oplus$
$E_2*f(I,L_2)$
≠
=
652
R_{I+1}:
$E_1*f(I+1,L_1)\oplus$
$E_2*f(I+1,L_2)$
≠
=
65X
R_{I+t-3}:
$E_1*f(I+t-3,L_1)\oplus$
$E_2*f(I+t-3,L_2)$
≠
=
660
$L(1)=L_1$ $E(1)=E_1$
$L(2)=L_2$ $E(2)=E_2$
$k = 2$
Figure 17
Figure 8

FIG. 6A

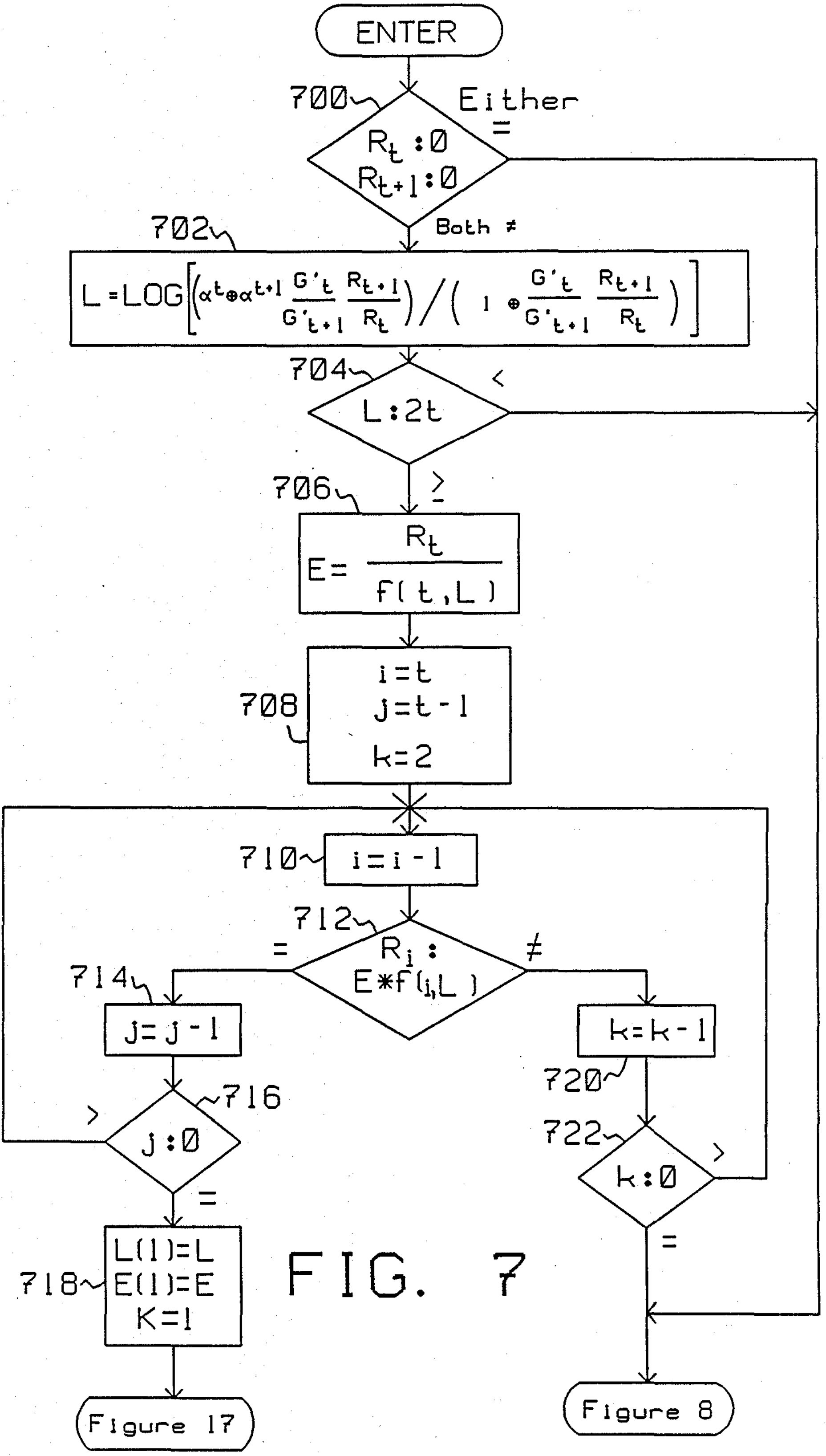

ENTER
700
$R_t : 0$
$R_{t+1} : 0$
Either =
Both ≠
702
$L = LOG\left[\left(\alpha^t \oplus \alpha^{t+1} \frac{G'_t}{G'_{t+1}} \frac{R_{t+1}}{R_t}\right) / \left(1 \oplus \frac{G'_t}{G'_{t+1}} \frac{R_{t+1}}{R_t}\right)\right]$
704
L:2t
<
≥
706
$E = \frac{R_t}{f(t,L)}$
708
i=t
j=t-1
k=2
710
i=i-1
712
R_i :
$E * f(i,L)$
=
≠
714
j=j-1
716
j:0
>
=
718
L(1)=L
E(1)=E
K=1
Figure 17
720
k=k-1
722
k:0
>
=
Figure 8

FIG. 7

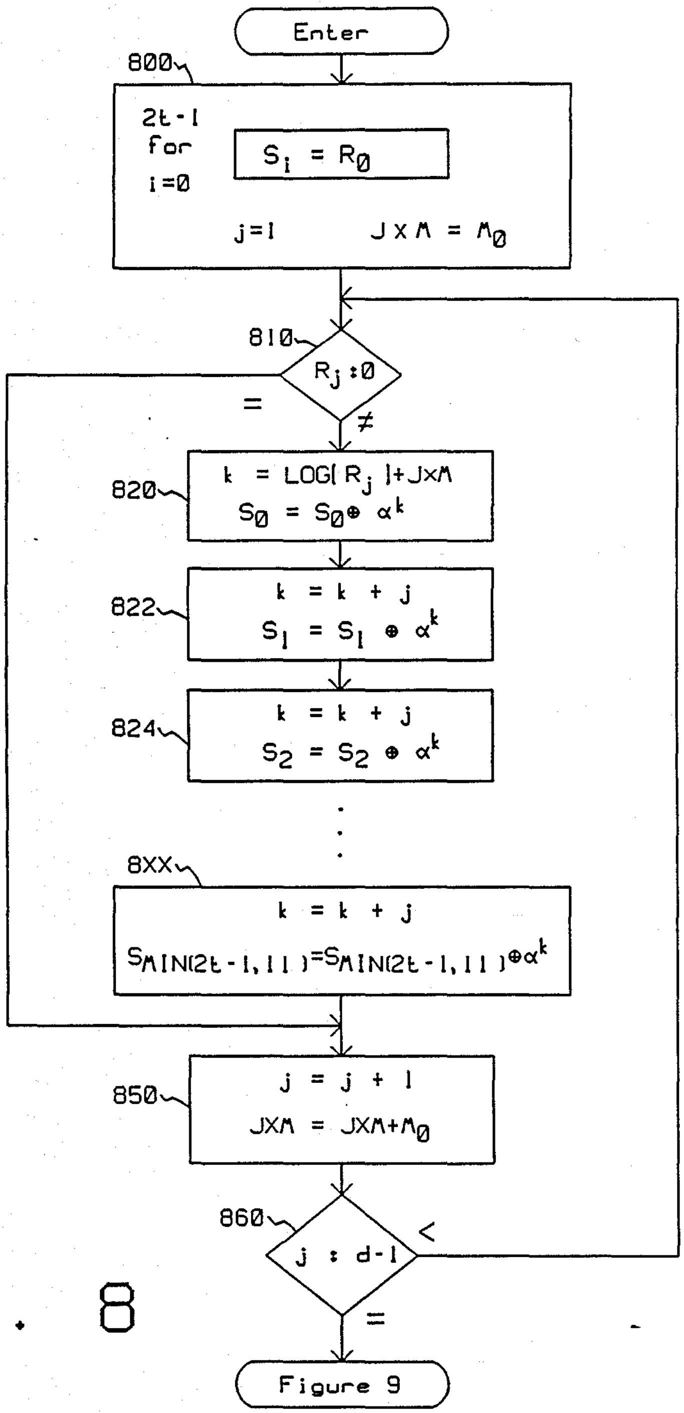

Enter
800
2t-1
for
i=0
$S_i = R_0$
j=1
$JXM = M_0$
810
$R_j : 0$
=
≠
820
$k = LOG[R_j]+JXM$
$S_0 = S_0 \oplus \alpha^k$
822
$k = k + j$
$S_1 = S_1 \oplus \alpha^k$
824
$k = k + j$
$S_2 = S_2 \oplus \alpha^k$
8XX
$k = k + j$
$S_{MIN(2t-1,11)} = S_{MIN(2t-1,11)} \oplus \alpha^k$
850
$j = j + 1$
$JXM = JXM + M_0$
860
$j : d-1$
<
=
Figure 9

FIG. 8

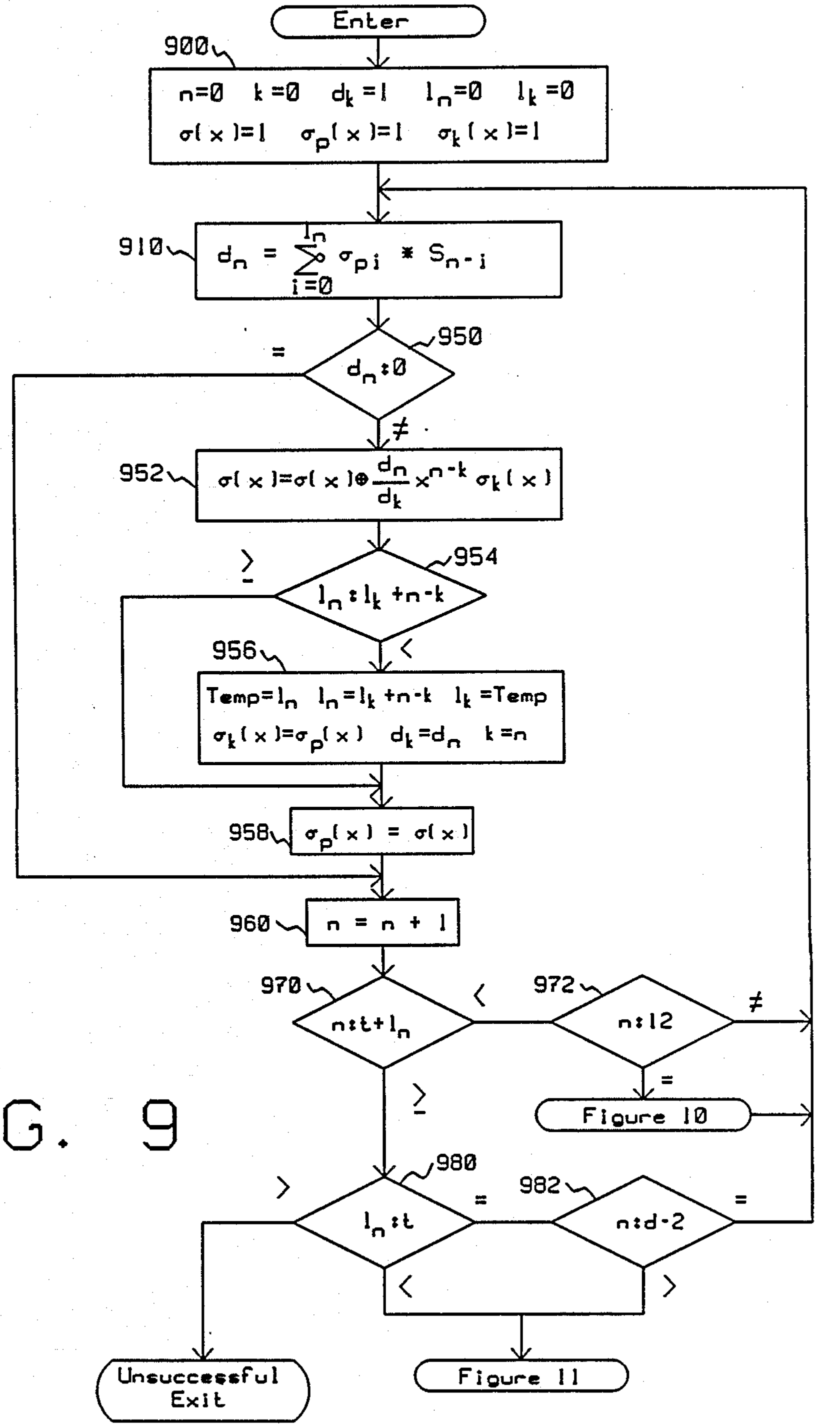

Enter
900
n=0 k=0 d_k=1 l_n=0 l_k=0
σ(x)=1 σ_p(x)=1 σ_k(x)=1
910
d_n = Σ_{i=0}^{l_n} σ_pi * S_n-i
950
d_n:0
=
≠
952
σ(x)=σ(x)⊕(d_n/d_k)x^(n-k) σ_k(x)
954
l_n:l_k+n-k
≥
<
956
Temp=l_n l_n=l_k+n-k l_k=Temp
σ_k(x)=σ_p(x) d_k=d_n k=n
958
σ_p(x) = σ(x)
960
n = n + 1
970
n:t+l_n
<
972
n:12
≠
=
Figure 10
≥
980
l_n:t
>
=
<
982
n:d-2
=
>
Unsuccessful Exit
Figure 11

FIG. 9

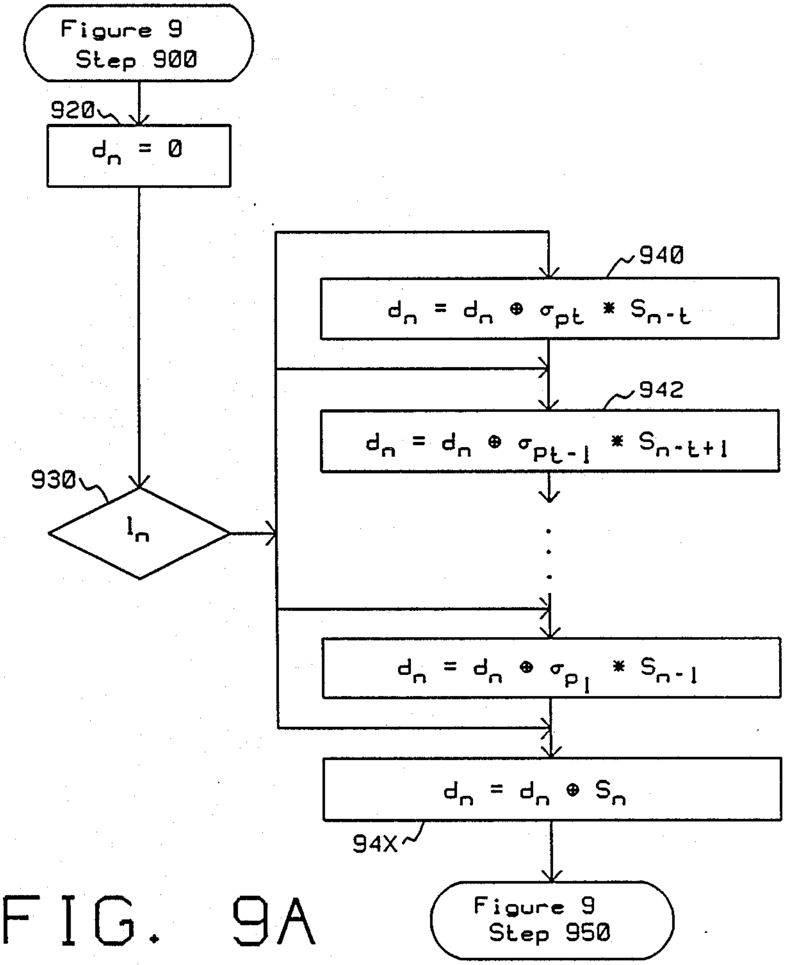


Figure 9
Step 900
920
$d_n = 0$
930
l_n
940
$d_n = d_n \oplus \sigma_{pt} * S_{n-t}$
942
$d_n = d_n \oplus \sigma_{pt-1} * S_{n-t+1}$
$d_n = d_n \oplus \sigma_{p_1} * S_{n-1}$
$d_n = d_n \oplus S_n$
94X
Figure 9
Step 950

FIG. 9A

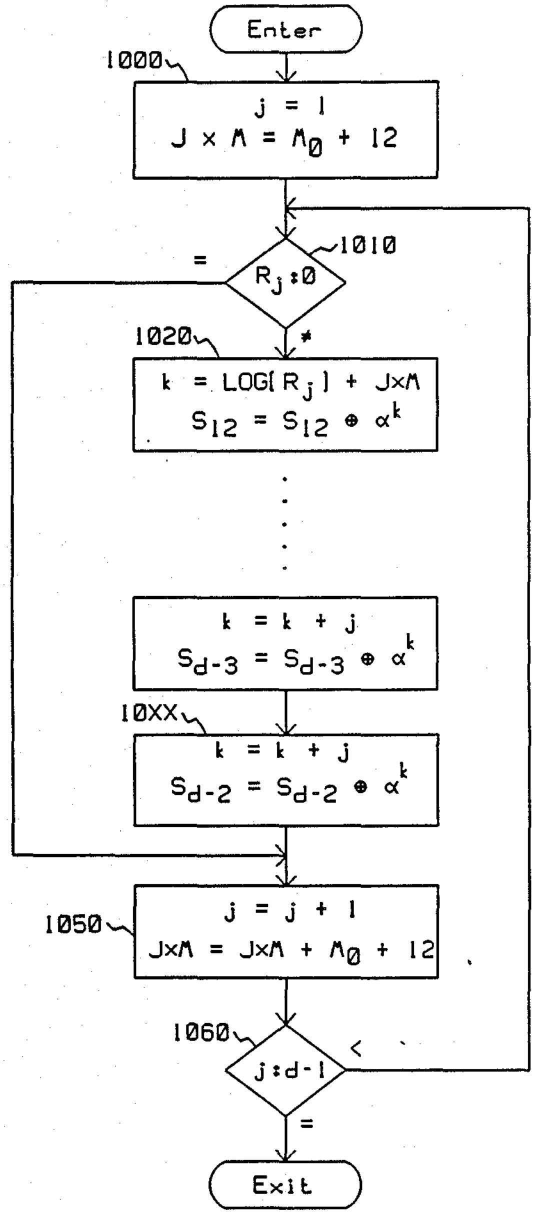

Enter
1000
j = 1
J × M = M$_0$ + 12
1010
=
R$_j$: 0
≠
1020
k = LOG[R$_j$] + J×M
S$_{12}$ = S$_{12}$ ⊕ α^k
k = k + j
S$_{d-3}$ = S$_{d-3}$ ⊕ α^k
10XX
k = k + j
S$_{d-2}$ = S$_{d-2}$ ⊕ α^k
1050
j = j + 1
J×M = J×M + M$_0$ + 12
1060
j : d-1
<
=
Exit

FIG. 10

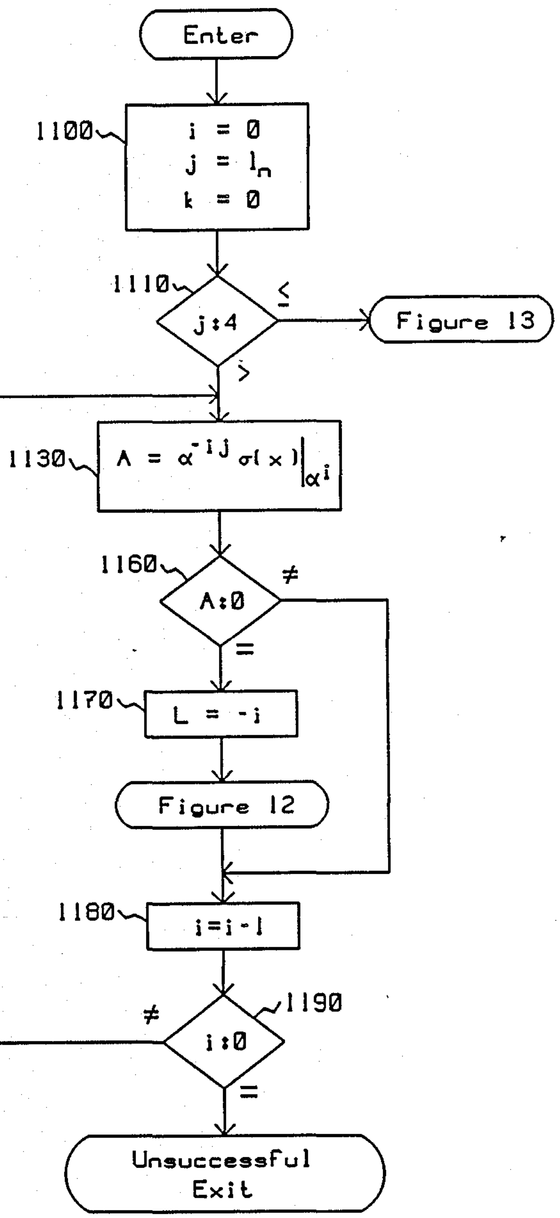

Enter
1100
i = 0
j = l_n
k = 0
1110
j:4
≤
Figure 13
>
1130
$A = \alpha^{-ij}\sigma(x)|_{\alpha^i}$
1160
A:0
≠
=
1170
L = -i
Figure 12
1180
i=i-1
1190
i:0
≠
=
Unsuccessful Exit

FIG. 11

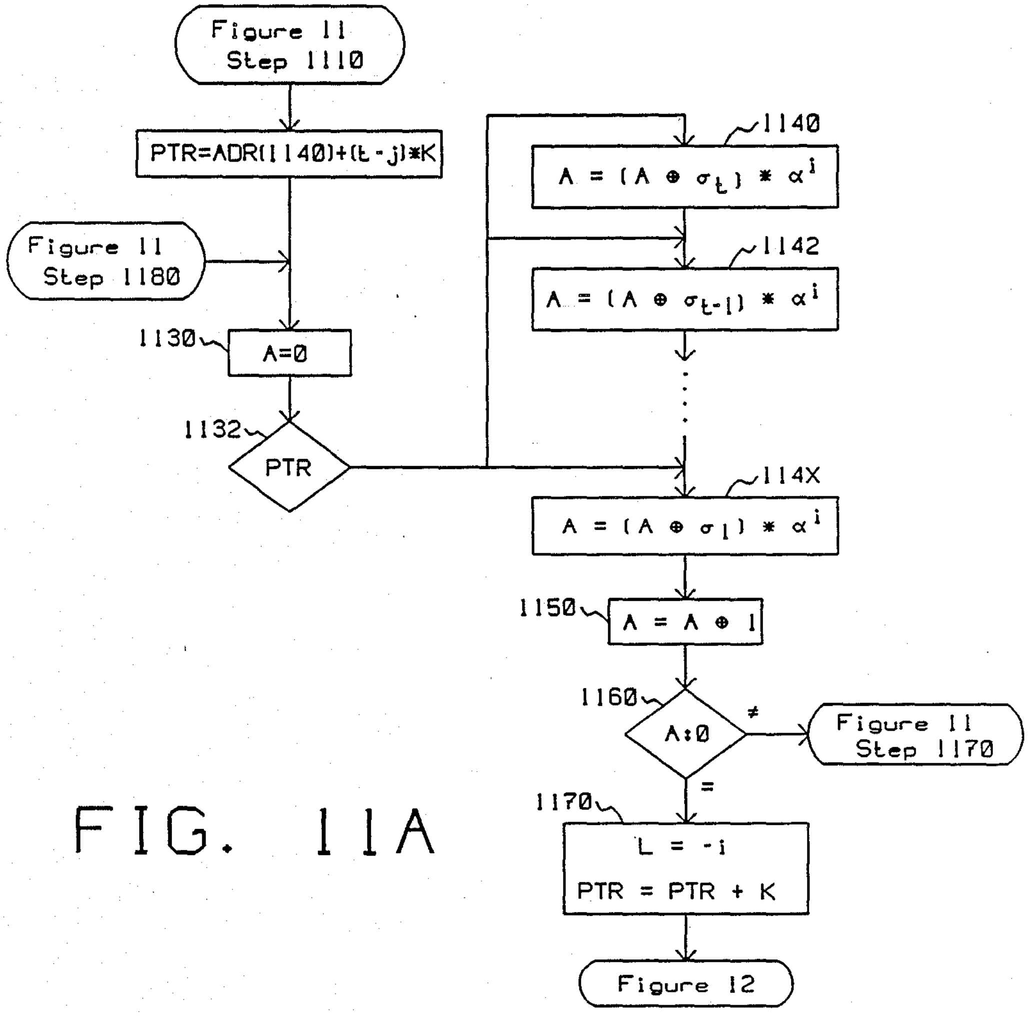


Figure 11
Step 1110
PTR=ADR(1140)+(t-j)*K
Figure 11
Step 1180
1130
A=0
1132
PTR
1140
A = (A ⊕ σ_t) * α^i
1142
A = (A ⊕ σ_t-1) * α^i
114X
A = (A ⊕ σ_1) * α^i
1150
A = A ⊕ 1
1160
A:0
≠
Figure 11
Step 1170
=
1170
L = -i
PTR = PTR + K
Figure 12

FIG. 11A

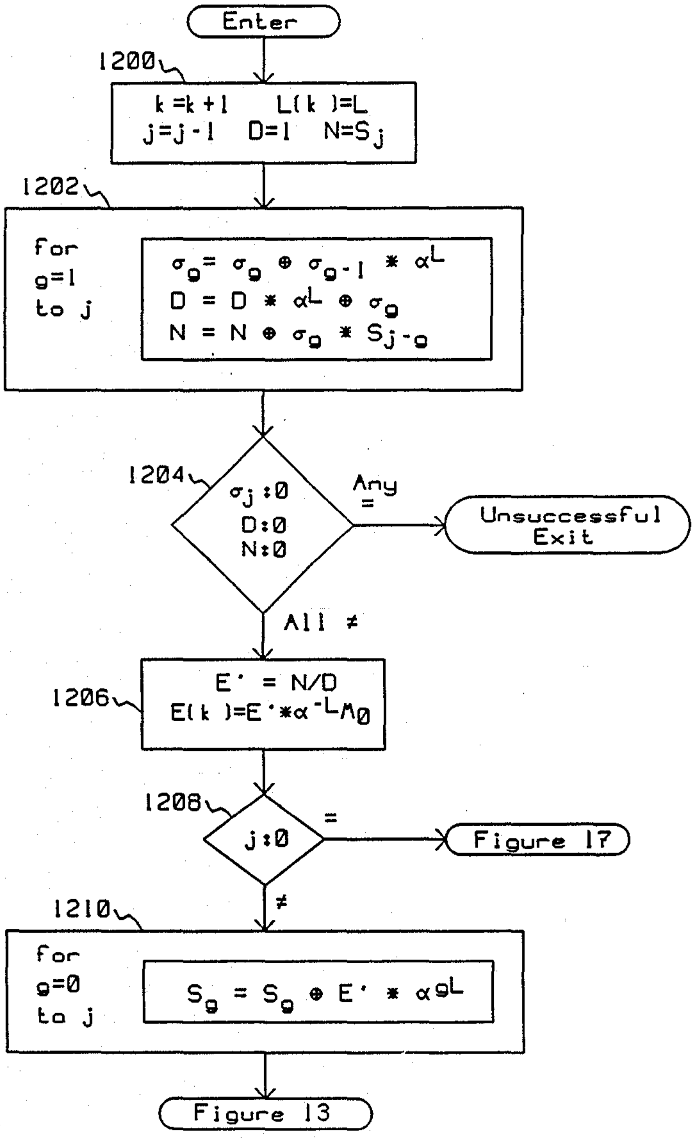

Enter
1200
k=k+1 L(k)=L
j=j-1 D=1 N=S_j
1202
for g=1 to j
$\sigma_g = \sigma_g \oplus \sigma_{g-1} * \alpha^L$
$D = D * \alpha^L \oplus \sigma_g$
$N = N \oplus \sigma_g * S_{j-g}$
1204
σ_j:0
D:0
N:0
Any =
Unsuccessful Exit
All ≠
1206
E' = N/D
$E(k) = E' * \alpha^{-L} M_0$
1208
j:0
=
Figure 17
≠
1210
for g=0 to j
$S_g = S_g \oplus E' * \alpha^{gL}$
Figure 13

FIG. 12

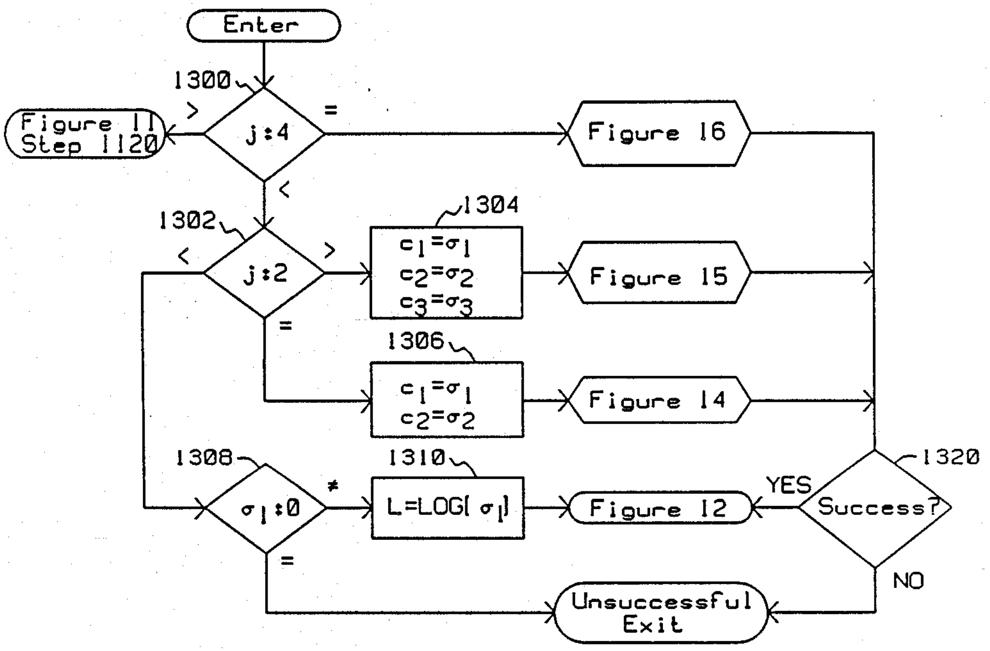

Enter
1300
j:4
>
=
<
Figure 11 Step 1120
Figure 16
1302
j:2
<
>
=
1304
$c_1=\sigma_1$
$c_2=\sigma_2$
$c_3=\sigma_3$
Figure 15
1306
$c_1=\sigma_1$
$c_2=\sigma_2$
Figure 14
1308
σ_1:0
≠
=
1310
L=LOG[σ_1]
Figure 12
YES
Success?
1320
NO
Unsuccessful Exit

FIG. 13

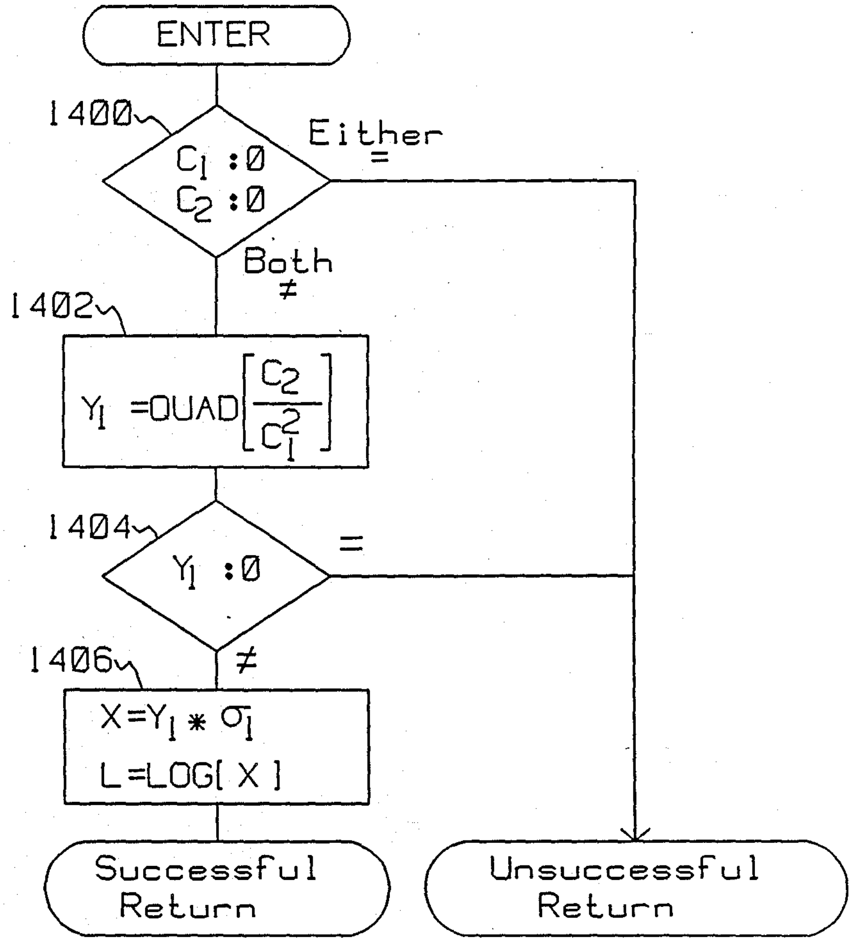

ENTER
1400
$C_1 : 0$
$C_2 : 0$
Either =
Both ≠
1402
$Y_1 = QUAD\left[\frac{C_2}{C_1^2}\right]$
1404
$Y_1 : 0$
=
≠
1406
$X = Y_1 * \sigma_1$
L=LOG[X]
Successful Return
Unsuccessful Return

FIG. 14

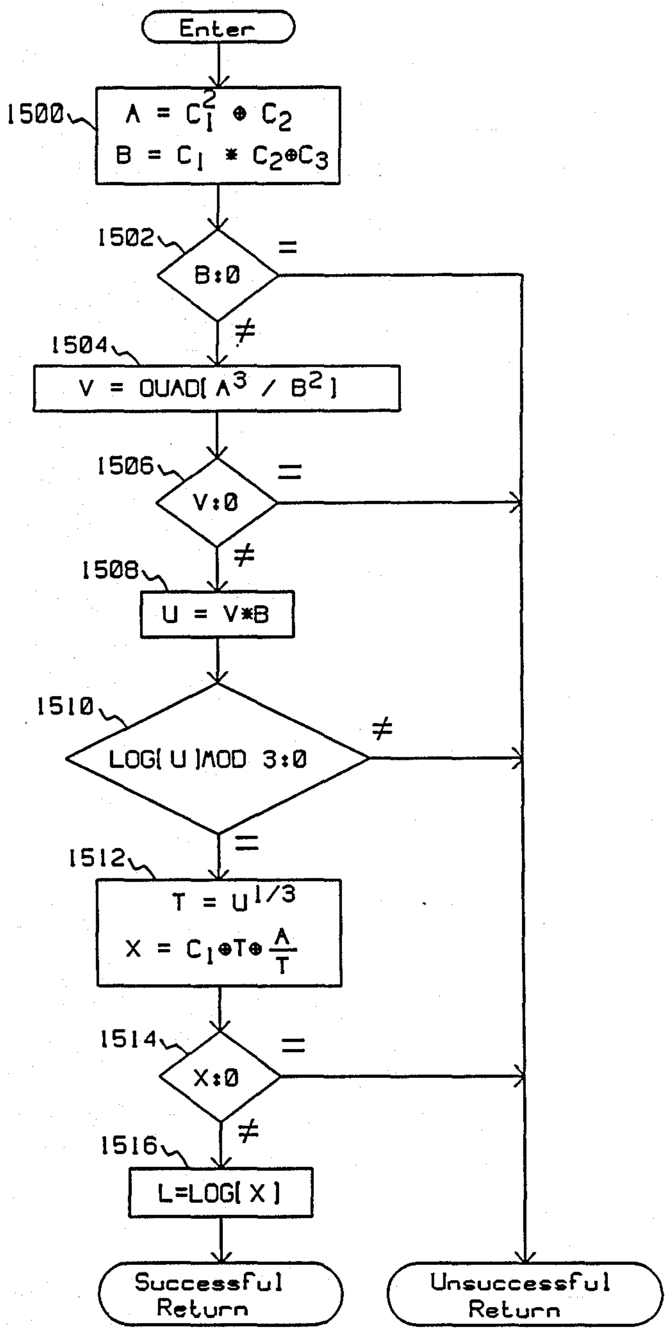

Enter
1500
A = C1² ⊕ C2
B = C1 * C2⊕C3
1502
B:0
=
≠
1504
V = QUAD[A³ / B²]
1506
V:0
=
≠
1508
U = V*B
1510
LOG[U]MOD 3:0
≠
=
1512
T = U^1/3
X = C1⊕T⊕A/T
1514
X:0
=
≠
1516
L=LOG[X]
Successful Return
Unsuccessful Return

FIG. 15

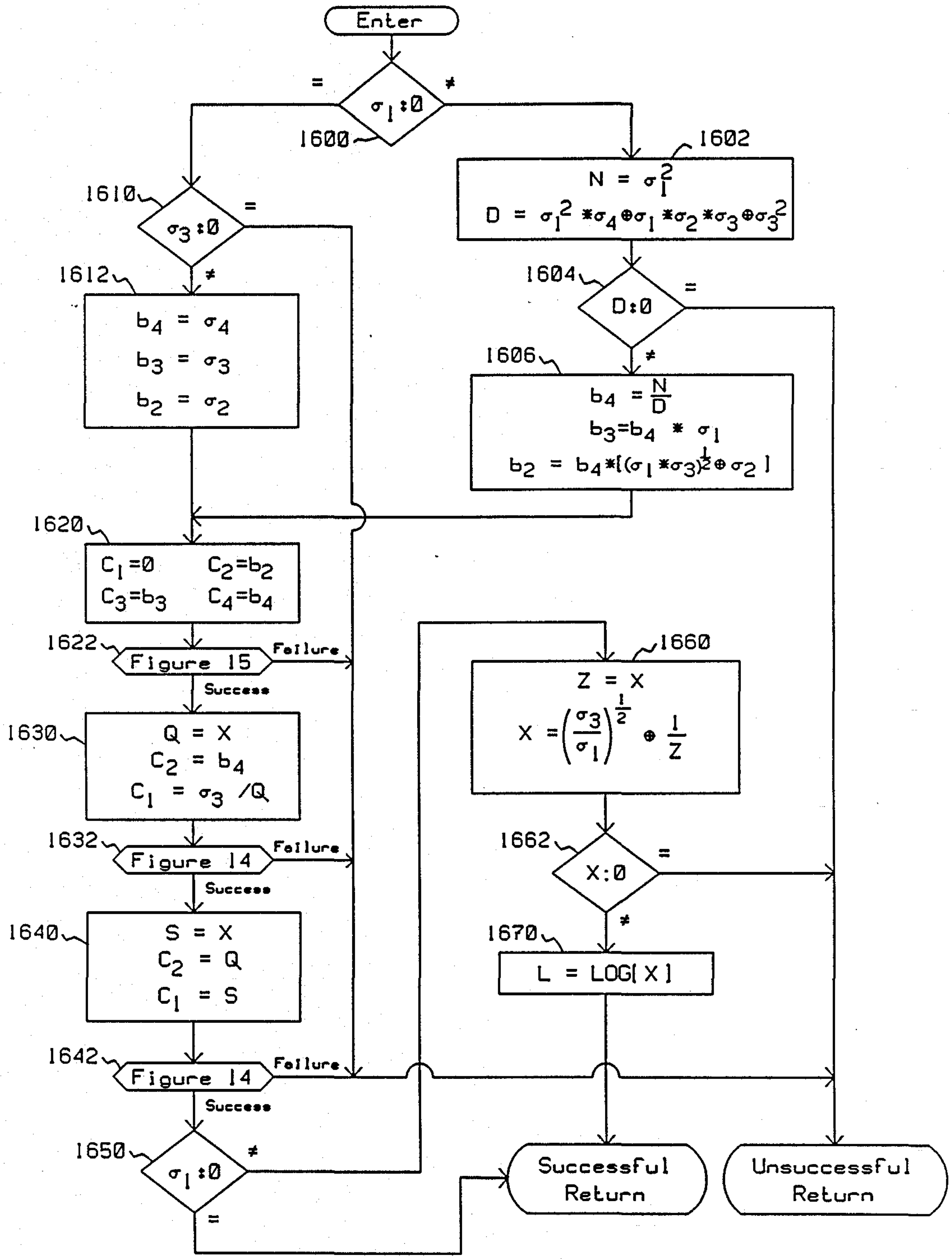

Enter
$\sigma_1 : 0$
1600
=
≠
1602
$N = \sigma_1^2$
$D = \sigma_1^2 * \sigma_4 \oplus \sigma_1 * \sigma_2 * \sigma_3 \oplus \sigma_3^2$
1610
$\sigma_3 : 0$
=
≠
1612
$b_4 = \sigma_4$
$b_3 = \sigma_3$
$b_2 = \sigma_2$
1604
D:0
=
≠
1606
$b_4 = \frac{N}{D}$
$b_3 = b_4 * \sigma_1$
$b_2 = b_4 * [(\sigma_1 * \sigma_3)^{\frac{1}{2}} \oplus \sigma_2]$
1620
$C_1 = 0$ $C_2 = b_2$
$C_3 = b_3$ $C_4 = b_4$
1622
Figure 15
Failure
Success
1660
$Z = X$
$X = \left(\frac{\sigma_3}{\sigma_1}\right)^{\frac{1}{2}} \oplus \frac{1}{Z}$
1630
$Q = X$
$C_2 = b_4$
$C_1 = \sigma_3 / Q$
1632
Figure 14
Failure
Success
1662
X:0
=
≠
1670
L = LOG[X]
1640
$S = X$
$C_2 = Q$
$C_1 = S$
1642
Figure 14
Failure
Success
1650
$\sigma_1 : 0$
≠
=
Successful Return
Unsuccessful Return

FIG. 16

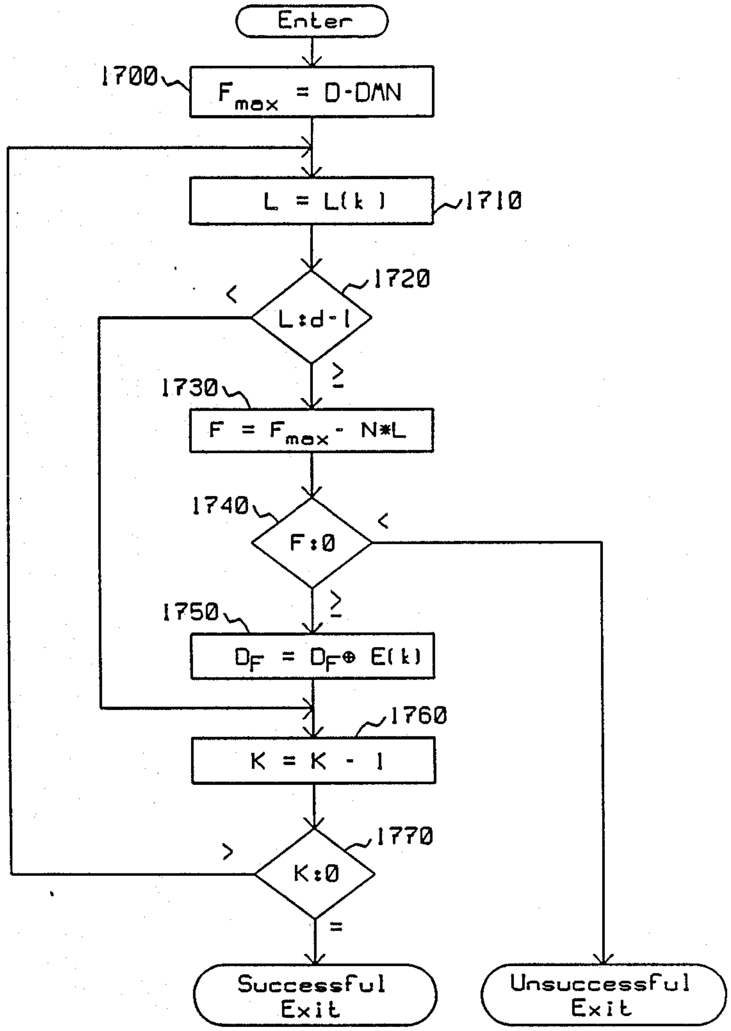

Enter
1700
F_{max} = D-DMN
L = L(k)
1710
1720
<
L:d-1
≥
1730
F = F_{max} - N*L
1740
<
F:0
≥
1750
D_F = D_F⊕ E(k)
1760
K = K - 1
1770
>
K:0
=
Successful Exit
Unsuccessful Exit

FIG. 17

FAST REMAINDER DECODING FOR A REED-SOLOMON CODE

BACKGROUND OF THE INVENTION

This invention relates to information storage and retrieval systems, and more particularly to means for decoding codewords for use in error detection and correction in such systems. Even more particularly, this invention relates to Reed-Solomon codes and related codes, and to improved methods for the decoding of such codes.

In a typical decoder, the coefficients of the syndrome polynomial, called frequency-domain syndromes, are first calculated by dividing the received codeword by each factor of the codé generator polynomial. This process is hardware-intensive and/or time-consuming, depending on implementation.

It is possible to efficiently generate the coefficients of the remainder polynomial, called time-domain syndromes, by dividing the received codeword by the entire code generator polynomial. The time-domain syndromes contain the same information as the frequency-domain syndromes.

Methods of decoding using frequency-domain syndromes are known in the prior art, for example see Chapter 3 Practical Error Correction Design for Engineers by Neal Glover and Chapter 5 of Error-Correction Coding for Digital Communications by Clark and Cain. For cases of one error, two frequency-domain syndromes can be used to generate an error location and value, which are then checked with other frequency-domain syndromes using a "Newton's Identities" test. A similar procedure starting with four frequency-domain syndromes may be used to decode cases of two errors. The calculation of frequency-domain syndromes adds significant additional hardware and/or time delay to decoding of cases of one and two errors.

For cases of more than two errors, the frequency-domain syndromes are typically used to iteratively generate an error locator polynomial and an error evaluator polynomial. The error locator polynomial is typically used in a "Chien" search to locate the symbols in error, and the error evaluator polynomial is used to calculate the values of the symbol errors. This process is inefficient when used to locate and evaluate errors in cases of less than five errors.

It is thus apparent that there is a need in the art for improved decoding methods for Reed-Solomon codes and related codes.

SUMMARY OF THE INVENTION

Accordingly, it is an object of the present invention to provide improved decoding methods which decrease the correction time required for cases of one and two errors by decoding directly from time-domain syndromes without generating frequency-domain syndromes.

Another object is to provide improved methods to decrease the correction time required for cases of more than two errors by eliminating the Chien search when the number of yet-to-be-located errors is or has been reduced to four or less.

Another object is to provide improved methods to decrease the correction time required for cases of more than two errors by calculating error values directly from the syndrome polynomial and error locator polynomial without generating the error evaluator polynomial.

Another object is to provide improved methods to decrease the correction time required for cases of more than two errors by reducing the degree of the error locator polynomial and adjusting the syndrome polynomial as each successive error is located.

Another object is to provide improved methods to decrease the correction time required for interleaved codewords of variable lengths by avoiding multiplications by maintaining a parameter for computing pointers to data symbols and remainder coefficients of the first such interleaved codeword and adjusting it as each successive codeword is corrected.

The above and other objects are attained by the present invention which comprises means and methods for fast decoding of Reed-Solomon and related codes. Cases of one and two symbols in error are decoded directly from the read remainder, without computing syndromes. The location of a single symbol in error is quickly determined using the result of a single finite field division as an index into a small precomputed table. Using another larger precomputed table, the value is calculated and the validity of the location and value are confirmed. The larger precomputed table is also used to calculate and validate the locations and values of two symbols in error. Fast decoding of cases of more than two symbols in error is accomplished by employing direct solution of an error locator polynomial to locate errors when four or fewer errors remain to be located and by employing an optimized Chien search to locate errors when more than four errors remain to be located. The invention further simplifies and speeds the decoding of all cases of more than two symbols in error by eliminating the need to compute an error evaluator polynomial and by adjusting the error locator and syndrome polynomials after each error is found.

The above and other objects, features, and advantages of the instant invention will be more apparent from the following more particular description thereof present in conjunction with the accompanying drawings.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 shows the environment in which the instant invention is used.

FIG. 2 illustrates the steps required to decode Cases B, C.1, and C.2.

FIG. 2A illustrates the steps required to validate the error location and value for Cases B, C.1, and C.2 using sequential repeated blocks.

FIG. 3 illustrates the steps required to calculate σ_1 and σ_2 for Case D using R_0 through R_3.

FIG. 4 illustrates the steps required to decode Case A and calculate σ_1 and σ_2 for Case D using R_{T-3} to R_T.

FIG. 5 illustrates the steps required to calculate the error locations and values for Case D.

FIG. 6 illustrates the steps required to validate the error locations and values for Case D.

FIG. 6A illustrates the steps required to validate the error locations and values for Case D using sequential repeated blocks.

FIG. 7 illustrates the steps required to decode Case C.3.

FIG. 8 illustrates the steps required to calculate the coefficients of S(x) required to decode cases of three and four errors for Case E.

FIG. **9** illustrates the steps required to iteratively generate the error locator polynomial for Case E.

FIG. **9**A illustrates the steps required to calculate the nth discrepancy d_n using sequential repeated blocks.

FIG. **10** illustrates the steps required to calculate the additional coefficients of S(x) required to decode cases of more than four errors for Case E.

FIG. **11** illustrates the steps required to locate and evaluate errors by searching for roots of $\sigma(x)$ for Case E.

FIG. **11**A illustrates the modifications to FIG. **11** required to evaluate $\alpha^{-ji}*\sigma(x)$ at $x=\alpha^i$ using sequential repeated blocks.

FIG. **12** illustrates the steps required to divide $\sigma(x)$ by $(x\oplus\alpha^L)$, compute the error value E, and adjust the coefficients of S(x) for Case E.

FIG. **13** illustrates the steps required to transfer control to the appropriate special error location subroutine for Case E.

FIG. **14** illustrates the steps required to solve for a root X, and its log L, of a quadratic equation in a finite field.

FIG. **15** illustrates the steps required to solve for a root X, and its log L, of a cubic equation in a finite field.

FIG. **16** illustrates the steps required to solve for the log L of one of the four roots of a quartic equation in a finite field.

FIG. **17** illustrates the steps required to correct errors in interleaved codewords.

DESCRIPTION OF THE PREFERRED EMBODIMENT

The following description is of the best presently contemplated mode of carrying out the instant invention. This description is not to be taken in a limiting sense but is made merely for the purpose of describing the general principles of the invention. The scope of the invention should be determined with reference to the appended claims.

In a finite field GF(2^m), elements are composed of m binary bits and addition ($\oplus$) consists of MODULO 2 summation of corresponding bits; this is equivalent to performing the bit-wise EXCLUSIVE-OR sum of operands:

$$x\oplus y=xXORy.$$

Note that subtraction is equivalent to addition since the MODULO 2 difference of bits is the same as their MODULO 2 sum.

Multiplication (*) may be implemented using finite field logarithm and antilogarithm tables wherein LOG $[\alpha^i]=i$ and A LOG $[i]=\alpha^i$:

$$\begin{aligned} x*y &= 0 && \text{if } x = 0 \text{ or } y = 0 \\ x*y &= \mathrm{ALOG}[\mathrm{LOG}[x] + \mathrm{LOG}[y]] && \text{if } x \neq 0 \text{ and } y \neq 0 \end{aligned}$$

where the addition of the finite field logarithms is performed MODULO 2^m-1. LOG [0] is undefined.

Division (/) may be implemented similarly:

$$\begin{aligned} &x/y \text{ is undefined} && \text{if } y = 0 \\ &x/y = 0 && \text{if } x = 0 \text{ and } y \neq 0 \\ &x/y = \mathrm{ALOG}[\mathrm{LOG}[x] - \mathrm{LOG}[y]] && \text{if } x \neq 0 \text{ and } y \neq 0. \end{aligned}$$

Note that for non-zero x, LOG $[1/x]=-$LOG $[x]=$LOG [x] XOR 2^m-1.

In a decoder for an error detection and correction system using a Reed-Solomon or related code of distance d for the detection and correction of a plurality of errors in codewords of n symbols comprised of $n-(d-1)$ data symbols and $d-1$ check symbols, each symbol an element of GF(2^m), a codeword C(x) is given by

$$C(x)=(x^{3-1}*I(x))\oplus((x^{d-1}*I(x)) \text{ MOD } G(x)) \quad (1)$$

where I(x) is an information polynomial whose coefficients are the $n-(d-1)$ data symbols and G(x) is the code generator polymomial

$$G(x) = \prod_{i=0}^{d-2} (x \oplus \alpha^{m0+i}) \quad (2)$$

where m_0 is a parameter of the code. A code of distance d can be used to correct all cases of $t=\mathrm{INT}((d-1)/2)$ errors without pointers and is guaranteed to detect all cases of INT(d/2) errors.

When e errors occur, the received codeword C'(x) consists of the EXCLUSIVE-OR sum of a transmitted codeword C(x) and the error polynomial E(x):

$$C'(x)=C(x)\oplus E(x) \quad (3)$$

where

$$E(x)=E_1*x^{L1}\oplus \ldots E_e*x^{Le} \quad (4)$$

L_i and E_i are the locations and values, respectively, of the e symbol errors.

The remainder

$$R(x)=R_{d-2}*x^{d-2}\oplus \ldots \oplus R_1*x\oplus R_0 \quad (5)$$

is given by

$$R(x)=C'(x) \text{ MOD } G(x) \quad (6)$$

that is, the remainder generated by dividing the received codeword C'(x) by the code generator polynomial G(x).

By equation (1),

$$C(x) \text{ MOD } G(x)=0 \quad (7)$$

so from equation (3),

$$R(x)=E(x) \text{ MOD } G(x) \quad (8)$$

For codes wherein $d\geqq 6$, the time required to decode cases of one and two errors may be reduced by decoding directly from the remainder by the use of a large pre-computed table f(i,L). The value of each element of f(i,L) is the coefficient of the x^i term of the remainder generated when x^L is divided by G(x), that is,

$$f(d-2,L)*x^{d-2}\oplus \ldots \oplus f(0,L)=x^L \text{ MOD } G(x) \quad (9)$$

Note that $f(i,L)\neq 0$ for all $i=0$ to $d-2$ and $L=d-1$ to 2^m-2.

Hereafter, expressions using elements of f(i,L) are written using the finite field antilogarithmic form of equation (9). Note however, that in the preferred embodiment of this invention the elements of f(i,L) are stored in finite field logarithmic form for computational efficiency. Without loss of generality, discussion is limited to codes wherein t≧3. To reduce the amount of storage required for the f(i,L) table, the one and two error correction procedure is structured to require only those elements of f(i,L) for L=d−1 to 2^m-2 and i=0 to T, where

$$T=2t-1 \text{ for } t=3,$$

$$T=\text{MAX}(7,t+1) \text{ for } t\geqq 4,$$

and only such elements are stored.

From equations (4), (8), and (9), a single error at location L with value E will generate a remainder with coefficients

$$R_i=E*f(i,L) \tag{10}$$

when e errors occur in a codeword, the coefficients of the remainder are the EXCLUSIVE-OR sum of the coefficients of the remainders for each error taken separately:

$$R_i = \sum_{j=1}^{e} o\ E_j*f(i,L_j). \tag{11}$$

CASE A

One or two check symbol errors

A.1: One error at location L<d−1 with value E. From equation (10), the remainder R(x) has a single non-zero coefficient

$$R_L=E.$$

A.2: Two errors at locations L_1<d−1 and L_2<d−1 with values E_1 and E_2. From equation (11), the remainder R(x) has two non-zero coefficients

$$R_{L1}E_1,$$

$$R_{L2}=E_2.$$

Cases A.1 and A.2 may be decoded by inspection of the remainder by counting the number of non-zero coefficients R_i.

In the preferred embodiment of this invention, nonzero coefficients R_i are counted for i=0 to t+1. If the total is less or equal to two, we are assured that there are at most d−t−1 errors, all at locations L_i<d−1, none of which must be corrected since none affects a data symbol. The time required to decode cases of one or more data errors is reduced by counting non-zero coefficients only after a coefficient equal to zero has been detected in the course of decoding Cases B, C, or D.

CASE B

One error at location L≧d−1 with value E. From equation (10), the coefficients of R(x) are given by

$$R_i=E*f(i,L).$$

For given a and b, a≠b, the ratio

$$\frac{f(b,L)}{f(a,L)}$$

is unique for each value of L such that d−1≦L<2^m−1. This yields a fast method for locating a single error. If either R_a=0 or R_b=0 there must be more than one error; otherwise the ratio R_b/R_a may be used as an index into a pre-computed location table $L_{a,b}$ derived according to

$$L_{a,b}\left[\frac{f(b,L)}{f(a,L)}\right]=L,$$

since from equation (10),

$$\frac{R_b}{R_a}=\frac{E*f(b,L)}{E*f(a,L)}=\frac{f(b,L)}{f(a,L)}.$$

Location table values corresponding to L<d−1 are set to an invalid number, for example zero, to flag the existence of more than one error.

In the preferred embodiment of this invention, the time required to determine the error location is reduced by using LOG [R_b/R_a] as an index into a location table $L_{a,b}$ derived according to

$$L_{a,b}\left[\text{LOG}\left[\frac{f(b,L)}{f(a,L)}\right]\right]=L.$$

When the location L is known, the error value E may be trivially computed from equation (10):

$$E=\frac{R_a}{f(a,L)}$$

The computed error location and value may be rapidly validated by gain using equation (10); test

$$R_i=E*f(i,L)$$

for values of i not equal to a or b, the indices of the coefficients R_i used to determine the error location and value.

In the preferred embodiment of this invention, R_0 and R_1 are used to determine L and E, and coefficients R_i are tested for i=2 to t. If all such coefficients are successfully tested, we are assured that there is one error at location L≦d−1 with value E and at most d−t−2 errors at locations $t+1<L_i<d-1$.

CASE C

Two errors, one error at location L_1≧d−1 with value E_1 and a second error at location L_2<d−1 with value E_2. From equation (11), the coefficients of R(x) are given by

$$R_i=E_1*f(i,L_1), \text{ for } i\neq L_2;$$

$$R_i=E_1*f(i,L_1)\oplus E_2, \text{ for } i=L_2. \tag{12}$$

In the preferred implementation of this invention, this case is divided into three subcases:

C.1: t+1<L_2<d−1

Decoding proceeds as in Case B: error location and value computations using R_0 and R_1 yield L_1 and E_1, and coefficients R_i are tested successfully for i=2 to t.

C.2: 2≦L_2≦t Error location and value computations proceed as in Case B, yielding L_1 and E_1, but the test of R_{L2} fails. If we perform one additional successful test of R_{t+1}, we are assured that there is one error at location

$L_1 \leqq d-1$ with value E_1 and at most $d-t-2$ errors at locations $2 < L_i < d-1$.

C.3: $0 \leqq L_2 \leqq 1$

If either R_0 or R_1 is equal to zero, the procedure of Case B detects more than one error. If both R_0 and R_1 are non-zero, the error location and value computations of Case B yield incorrect results and more than one test of the coefficients R_i fails. In either case we branch to Case D.

If either R_0 or R_1 is equal to zero and the computations of Case D are restricted to non-zero coefficients, the computation of the denominator of the coefficients of $\sigma(x)$ using $R_{T\text{-}3}$ to R_T will produce zero. If both R_0 and R_1 are non-zero or the computations of Case D are not restricted to non-zero coefficients, the error location computations of Case D using R_0 to R_3 will correctly locate the two errors, with one having location $0 \leqq L_i < 1$. In either case, we may then repeat the single error location and value computations of Case B but using R_t and R_{t+1} in place of R_0 and R_1, determining L using from a location table $L_{t,t+1}$ derived from $f(t+1,L)/f(t,L)$), and validating the results by testing the coefficients R_i for $i=0$ to $t-1$, allowing one failure at $i=0$ or $i=1$.

Case C.3 occurs with sufficiently low frequency that table storage space may be reduced without signficantly degrading performance by eliminating the $L_{t,t+1}$ table and determining the location of the data symbol error by calculating L as a function of R_{t+1}/R_t:

$$L = \text{LOG}\left[\frac{\alpha^t \oplus \alpha^{t+1} * \dfrac{G_t'}{G_{t+1}'} * \dfrac{R_{t+1}}{R_t}}{1 \oplus \dfrac{G_t'}{G_{t+1}'} * \dfrac{R_{t+1}}{R_t}}\right] \tag{13}$$

where G_i' is the coefficient of the x^i term of

$$G'(x) = \prod_{i=0}^{d-3} (x \oplus \alpha^{m_0+i})$$

In the preferred implementation of this invention, L for Case C.3 is computed from equation (13) using $a=t$ and $b=t+1$. One validation testing failure is allowed for any $i=0$ to $t-1$. If validation testing succeeds, we are assured that there is one error at location $L_1 > d-1$ with the value E_1, a second error at location $0 \leqq L_2 \leqq t-1$, and at most $d-t-3$ errors at locations $t+2 \leqq L_i < d-1$.

CASE D

Two errors at locations $L_1 > d-1$ and $L_2 \geqq d-1$ with values E_1 and E_2. From equation (11), the coefficients of the remainder R(x) are given by

$$R_i = E_1 * f(i,L_1) \oplus E_2 * f(i,L_2) \tag{14}$$

The coefficients $\sigma_1 = \alpha^{L1} \oplus \alpha^{L2}$ and $\sigma_2 = \alpha^{L1} * \alpha^{L2}$ of the error locator polynomial $\sigma(x)$ may be computed from four coefficients R_a, R_b, R_c, and R_d according to the equations

$$\sigma_1 = \frac{A_{ab}R_aR_b \oplus A_{ac}R_aR_c \oplus A_{ad}R_aR_d \oplus A_{bc}R_bR_c \oplus A_{bd}R_bR_d \oplus A_{cd}R_cR_d}{C_{ab}R_aR_b \oplus C_{ac}R_aR_c \oplus C_{ad}R_aR_d \oplus C_{bc}R_bR_c \oplus C_{bd}R_bR_d \oplus C_{cd}R_cR_d}$$

$$\sigma_2 = \frac{B_{ab}R_aR_b \oplus B_{ac}R_aR_c \oplus B_{ad}R_aR_d \oplus B_{bc}R_bR_c \oplus B_{bd}R_bR_d \oplus B_{cd}R_cR_d}{C_{ab}R_aR_b \oplus C_{ac}R_aR_c \oplus C_{ad}R_aR_d \oplus C_{bc}R_bR_c \oplus C_{bd}R_bR_d \oplus C_{cd}R_cR_d}$$

where the pre-computed constants A_{ab}, B_{ab}, C_{ab}, etc. are functions of a, b, c, and d of the form

$$A_{ab} = (\alpha^{2a} \oplus \alpha^{2b}) * (G''_{c-1} * G_d'' \oplus G_c'' * G''_{d-1}),$$

$$B_{ab} = (\alpha^{a} * \alpha^{2b} \oplus \alpha^{b} * \alpha^{2a}) * (G''_{c-1} * G_d'' \oplus G_c'' * G''_{d-1}).$$

$$C_{ab} = (\alpha^{a} \oplus \alpha^{b}) * (G''_{c-1} * G_d'' \oplus G_c'' * G''_{d-1}).$$

and G_i'' is the coefficient of the x^i term of

$$G''(x) = \prod_{i=0}^{d-4} (x \oplus \alpha^{m_0+i}),$$

with G_{-1}'' defined as zero.

In the preferred embodiment of this invention, R_0 to R_3 are used in computing σ_1 and σ_2. For codes wherein $t \geqq 4$, the time required to compute σ_1 and σ_2 is reduced by choosing four non-zero zero coefficients of R(x): if any of R_0 to R_3 is equal to zero, σ_1 and σ_2 are computed using $R_{T\text{-}3}$ to R_T. If any of $R_{T\text{-}3}$ to R_T is also equal to zero, there exist more than two errors.

We now have the coefficients of the error locator polynomial for the case of two errors

$$x^2 \oplus \sigma_1 * x \oplus \sigma_2 = 0.$$

Solution of a quadratic equation in a finite field is known in the prior art; for example, see Chapter 3 of Practical Error Correction Design for Engineers by Neal Glover. Substituting $x = y * \sigma_1$ yields

$$y^2 \oplus y \oplus c = 0, \text{ where } c = \frac{\sigma_2}{\sigma_1^2}.$$

For each odd solution to this equation Y_1, there is an even solution $Y_2 = Y_1 \oplus 1$. Y_1 can be fetched from a pre-computed quadratic table derived according to

$$\text{QUAD}[i^2 \oplus i] = i \oplus 1 \text{ for } i = 0, 2, \ldots 2^m - 2$$

using c as an index. There are 2^{m-1} such pairs of solutions; the other elements of the table are set to an invalid number, for example zero, to flag the existence of more than two errors. When $Y_1 \neq 0$ and $Y_2 \neq 0$ have been determined, reverse substitution yields expressions for the two error locations

$$L_1 = \text{LOG}\,[\sigma_1 * Y_1]$$

$$L_2 = \text{LOG}\,[\sigma_1 * Y_2]$$

If both $L_i < d-1$ and neither L_i is not equal to a or b, the indices of the coefficients R_i used in Case B, there must exist more than two errors, otherwise the error situation of Case C.3 may exist.

In the preferred implementation of this invention, Case C.3 is attempted if $L_i < d-1$, whether or not L_i is equal to a or b; Case C.3 will detect cases of more than two errors.

Solving the system of equations

$$R_a=E_1*f(a,L_1)\oplus E_2*f(a,L_2)$$

$$R_b=E_1*f(b,L_1)\oplus E_2*f(b,L_2)$$

yields expressions for the error values

$$E_1=\frac{R_a*f(b,L_2)\oplus R_b*f(a,\ L_2)}{f(a,L_1)*f(b,L_2)\oplus f(b,L_1)*f(a,L_2)}$$

$$E_2=\frac{R_a*f(b,L_1)\oplus R_b*f(a,L_1)}{f(a,L_1)*f(b,L_2)\oplus f(b,L_1)*f(a,L_2)}$$

In the preferred embodiment of the invention, R_0 and R_1 are used in calculating error values if R_0 to R_3 were used in determining σ(x), while $R_{T\text{-}3}$ and $R_{T\text{-}2}$ are used in calcuating error values if $R_{T\text{-}3}$ to R_T were used in determining σ(x).

The computed error locations and values may be rapidly validated using equation (11); test

$$R_i=E_1*f(i,L_1)\oplus E_2*f(i,L_2)$$

for values of i not equal to a, b, c, or d, the indices of the coefficients R_i used to determine the error locations and values.

In the preferred embodiment of this invention, coefficients R_i are tested for i=4 to t+1 if R_0 to R_3 were used in determining locations and values, and coefficients R_i are tested for i=0 to T-4 if $R_{T\text{-}3}$ to R_T were used. If all such coefficients are successfully tested, we are assured that there are two errors at locations $L_1>d-1$ and $L_2>d-1$ with values E_1 and E_2 and at most $d-t-3$ errors at locations $t+2\leqq L_i\leqq d-1$.

CASE E

More than two errors, at locations L_i with values E_i.

SYNDROME POLYNOMIAL GENERATION: We compute the coefficients S_i of the frequency-domain syndrome polynomial

$$S(x)=S_{d-2}*x^{d-2}\oplus\ldots\oplus S_1*x\oplus S_0$$

from the coefficients R_i of the remainder R(x) according to

$$S_i=\sum_{j=0}^{d-2}R_j*\alpha^{j(m0+i)}.$$

Sequential computation of each coefficient S_i would require d−1 references to each coefficient R_j. Physical constraints and interleaving of multiple codewords often make each reference to a coefficient R_j difficult and time-consuming.

In the preferred embodiment of this invention, the time required to calculate the coefficients of S(x) is reduced by the following methods:

(1) Each coefficient R_j is referenced once and its contribution to each coefficient S_i is sequentially computed and added.

(2) A partial result for the contribution of each coefficient R_j to each coefficient S_i is maintained in finite field logarithmic form;

(3) The use of software loops is minimized by using repeated sequential blocks.

(4) The calculation of coefficients S_i required for the decoding of cases of more than four errors is deferred until the iterative error locator polynomial generation routine determines the existence of more than four errors.

ERROR LOCATOR POLYNOMIAL GENERATION: The coefficients of S(x) are used to iteratively generate the coefficients of the error locator polynomial σ(x). Such iterative algorithms are known in the prior art; for example, see Chapter 5 of Error-Correction Coding for Digital Communications by Clark and Cain.

In the preferred embodiment of this invention, the time required to evaluate each successive discrepancy value is reduced by using the current degree of σ(x) as an index into a table of software jump addresses of appropriate starting points for each evaluation in a sequence of repeated blocks.

ERROR LOCATION AND EVALUATION: If the degree of σ(x) indicates more than four errors exist, we evaluate σ(x) at $x=\alpha^L$ for each L, $0\leqq L<2^m-1$, until the result is zero, which signifies that α^L is a root of σ(x) and L is an error location.

In the preferred embodiment of this invention, the time required to evaluate σ(x) at each successive location is reduced by maintaining a software address pointer to the appropriate starting point for each evaluation in a sequence of repeated blocks, and by evaluating $\alpha^{jL}*\sigma(x)$ at $x=\alpha^{-L}$ where j is the degree of σ(x), rather than evaluating σ(x) at $x=\alpha^L$.

When the location L of an error has been determined, σ(x) is divided by $(x\oplus\alpha^L)$, producing a new error locator polynomial of degree one less than that of the old:

$$\sigma(x)=\frac{\sigma(x)}{x\oplus\alpha^L}.$$

The error value E may be calculated directly from S(x) and the new σ(x) using

$$E=\alpha^{-Lm0}*\frac{\sum_{i=0}^{j}\sigma_i*S_{j-i}}{\sigma(x)\quad\alpha^L}$$

where j is the degree of the new σ(x).

In the preferred embodiment of this invention, the division of σ(x) by $(x\oplus\alpha^L)$ and the calculation of the numerator and denominator of E are all performed in a single software loop.

When the location L and value E of an error have been determined, the coefficients of S(x) are adjusted to remove its contribution according to

$$S_i=S_i\oplus E*\alpha^{L(m0+i)}$$

By reducing the degree of σ(x) and adjusting S(x) as the location and value of each error are determined, the time required to locate and evaluate each successive error is reduced.

When the degree of j of σ(x) is four or less, the time required to locate the remaining errors is reduced by using the special error locating routines below, each of which locates one of the remaining errors without using the Chien search. After the location of an error has been determined by one of the special error locating routines, its value is calculated σ(x) is divided by $(x\oplus\alpha^L)$, and S(x) is adjusted in the same way as when an error is located by evaluating σ(x).

When j=1, the error locator polynomial is

$$x \oplus \sigma_1 = 0$$

By inspection, the root of this equation is $\dot{\sigma}_1 = \alpha^L$. Thus

$$L = \text{LOG}\,[\sigma_1].$$

When j=2, the error locator polynomial is

$$x^2 \oplus \sigma_1 * x \oplus \sigma_2 = 0.$$

One of the roots of this equation may be found using the quadratic method of Case D:

$$L = \text{LOG}\left[\sigma_1 * \text{QUAD}\left[\frac{\sigma_2}{\sigma_1^2}\right]\right].$$

When j=3, the error locator polynomial is

$$x^3 \Gamma \sigma_1 * x^2 \oplus \sigma_2 * x \oplus \sigma_3 = 0.$$

Solution of a cubic equation in a finite field is known in the prior art; for example, see Flagg, U.S. Pat. No. 4,099,162. Substituting

$$x = w \oplus \sigma_1,\ w = t \oplus B/t, \text{ and } v = t/B^3/B$$

yields a quadratic equation in v:

$$v^2 \oplus v \oplus A^3/B^2 = 0$$

where

$$A = \sigma_1^2 \oplus \sigma_2 \text{ and } B = \sigma_1 * \sigma_2 \oplus \sigma_1.$$

A root V of this equation may be found by the quadratic method above. Then by reverse substitution

$$L = \text{LOG}\left[\sigma_1 \oplus (B*V)^{\frac{1}{3}} \oplus \frac{A}{(B*V)^{\frac{1}{3}}}\right].$$

When j=4, the error locator polynomial is

$$x^4 \oplus \sigma_1 * x^3 \oplus \sigma_2 * x^2 \oplus \sigma_3 * x \oplus \sigma_4 = 0.$$

Solution of a quartic equation in a finite field is known in the prior art; for example, see Deodhar, U.S. Pat. No. 4,567,594. If $\sigma_1 = 0$, assign $b_i = \sigma_i$ for i=2 to 4, otherwise substitute

$$z = \frac{1}{x \oplus (\sigma_3/\sigma_1)^{\frac{1}{2}}}$$

to give

$$z^4 \oplus b_2 z^2 \oplus b_3 z \oplus b_4 = 0$$

where

$$b_4 = \frac{\sigma_1^2}{\sigma_1^{2} * \sigma_4 \oplus \sigma_1 * \sigma_2 * \sigma_3 \oplus \sigma_3^2},$$

$$b_3 = \sigma_1 * b_4,$$

$$b_2 = ((\sigma_1 * \sigma_3)^{\frac{1}{2}} \oplus \sigma_2) * b_4.$$

The resulting affine polynomial may be solved in the following manner:

(1) Solve for a root Q of the equation $q^3 \oplus b_2 * q \oplus b_3 = 0$ by the cubic method above.

(2) Solve for a root S of the equation $s^2 \oplus b_3/Q * s \oplus b_4 = 0$ by the quadratic method above.

(3) Solve for a root Z of the equation $z^2 \oplus Q * z \oplus S = 0$ by the quadratic method above. If $\sigma_1 = 0$, L=LOG [Z], otherwise reverse substitution yields

$$L = \text{LOG}\left[(\sigma_3/\sigma_1)^{\frac{1}{2}} \oplus \frac{1}{Z}\right]$$

Referring to FIG. **1**, a data controller **100** having a host interface **102** is connected to a host computer **104**. The data controller **100** also has a device interface **101** which connects the data controller **100** to an information storage device **108**.

In the process of reading data from information storage device **108**, data bits from information storage device **108** are transferred through device information channel **116**, through the device interface **101**, and into the decoder **110**. At the same time the data bits are being transferred into the encoder and time domain syndrome generator **110**, they are transferred in parallel into the data buffer **106**. As check bits are transferred into the decoder **110**, remainder bits are generated by the decoder **110** and transferred to the remainder buffer **107**. After the data bits have been transferred into the data buffer **106** and remainder bits transferred to the remainder buffer **107**, the processor **105** uses the remainder bits from the remainder buffer **107** to detect and correct, if necessary, errors in the data bits in data buffer **106**. After correction of any errors in the data buffer **106**, the data bits are transferred through the host interface **102**, through the information channel **118** through the host computer **104**.

Referring to FIG. **2**, if either R_0 or R_1 is equal to zero, Step **200** transfers control to FIG. **4**, having detected more than one error. Otherwise Step **202** determines the location $^{L=L}$**0,1**[R_1/R_0]. If L is invalid, that is, less than d−1, Step **204** transfers control to FIG. **3**. Otherwise Step **206** calculates the error value E=R_0/f(O,L). Step **210** prepares to validate L and E by initializing counters i=1, j=t−1, and k=2. Step **212** increments counter i. If $R_i \neq E*f(i,L)$, Step **214** transfers control to Step **220**. Otherwise Step **216** decrements counter j. If counter j is then still greater than zero, step **218** transfers control back to Step **212** to continue testing coefficients R_i. Otherwise L and E have been validated and Step **250** records them before transferring control to FIG. **17** for error correction. After a coefficient R_i fails the test of Step **214**, Step **220** decrements counter k. If counter k is then still greater than zero, Step **222** transfers control back to Step **212** to continue testing coefficients R_i. Otherwise L and E are invalid and control is transferred to FIG. **3**.

The time required for executing looping control and maintaining counters in FIG. **2** may be eliminated by replacing Steps **210** to **222** of FIG. **2** with the repeated sequential blocks of FIG. 2A. Steps **230** through **23**x sequentially test coefficients R_i for i=2 to t. If all of these tests succeed, L and E have been validated and Step **23**x transfers control to Step **250** of FIG. **2**. Otherwise control is transferred into Steps **240** to **24**x, where sequential testing of coefficients R_i continues through i=t+1. If all of the remaining tests in this second set succeed, L and E have been validated and Step **24**x transfers control to Step **250** of FIG. **2**. If any of the tests of Steps **240** to **24**x fails, L and E are invalid and control is transferred to FIG. **3**.

Referring to FIG. **3**, on entry both R_0 and R_1 are known to be non-zero. If $t \geqq 4$ and either R_2 or R_3 is equal to zero, Step **300** transfers control to FIG. **4** to attempt calculation of σ_1 and σ_2 using $R_{T\text{-}3}$ through R_T. Otherwise Step **302** calculates the products of R_0 to R_3 taken two at a time. Step **304** computes the denominator of σ_1 and σ_2. If the denominator is equal to zero, Step **306** transfers control to FIG. **8**. Otherwise Step **308** computes the log of the denominator, and the numerator of σ_1. If the numerator of σ_1 is equal to zero, Step **310** transfers control to FIG. **8**. Otherwise Step **312** computes the log of σ_1, and the numerator of σ_2. If the numerator of σ_2 is equal to zero, Step **314** transfers control to FIG. **8**. Otherwise Step **316** calculates the log of σ_2 and sets counter I=0 to signify that R_0 to R_3 were used in computing σ_1 and σ_2, then transfers control to FIG. **5**.

Referring to FIG. **4**, on entry at least one of R_0 to R_3 is known to be equal to zero. Step **400** counts the number of non-zero coefficients R_i for i=0 to t+1. If the result is less than three, there are no data symbols in error; Step **402** exits the correction procedure successfully without correcting any data symbols, having decoded Case A. Otherwise if any of $R_{T\text{-}3}$ to R_T are equal to zero, Step **404** transfers control to FIG. **8**, having detected more than two errors. Otherwise Step **406** calculates the products of $R_{T\text{-}3}$ to R_T taken two at a time. Step **408** computes the denominator of σ_1 and σ_2. If the denominator is equal to zero, Step **410** transfers control to FIG. **7**, having possibly detected Case C.3. Otherwise Step **412** computes the log of the demoninator, and the numerator σ_1. If the numerator of σ_1 is equal to zero, Step **414** transfers control to FIG. **8**. Otherwise Step **416** computes the log of σ_1, and the numerator of σ_2. If the numerator of σ_2 is equal to zero, Step **418** transfers control to FIG. **8**. Otherwiste STep **420** calculates the log of σ_2 and sets counter $I=T-3$ to signify that $R_{T\text{-}3}$ to R_T were used in computing σ_1 and σ_2, then transfers control to FIG. **5**.

In FIG. **5**, which computes the error locations and values for Case D, Step **500** calculates the quantity $c=\sigma_2/(\sigma_1)^2$ and fetches $Y_1=QUAD[c]$; Y_1 is known to be not equal to one, since σ_2 is known to be non-zero. If Y_1 is equal to zero, Step **502** transfers control to FIG. **8**. Otherwise Step **504** calculates locations L_1. If L_1 is less than d−1, Step **506** transfers control to FIG. **7**, having possibly detected Case C.3. Otherwise Step **508** calculates Y_2 and location L_2; Y_2 is known to be not equal to zero since Y_1 is not equal to one. If L_2 is less than d−1, Step **510** transfers control to FIG. **7**, having possibly detected Case C.3. Otherwise Step **514** computes the denominator of error values E_1 and E_2. If the denominator is equal to zero, Step **516** transfers control to FIG. **8**. Otherwise Step **518** calculates the numerator of E_1. If the numerator of E_1 is equal to zero, Step **520** transfers control to FIG. **8**. Otherwise Step **522** calculates the numerator of E_2. If the numerator of E_2 is equal to zero, Step **524** transfers control to FIG. **8**. Otherwise Step **526** calculates E_1 and E_2, then transfers control to FIG. **6**.

Referring to FIG. **6**, Step **600** initializes counter i=4. If counter I is equal to zero, Step **602** transfers control to Step **606**. Otherwise Step **604** initializes counter i=0. Step **606** initializes counter j=t−2. If $R_i \neq E_1*f(i,L_1.)\oplus E_2*f(i,L_2)$, the computed error locations and values are invalid and Step **608** transfers control to FIG. **8**. Otherwise Step **610** increments counter i and decrements counter j. If counter j is then still greater than zero, step **612** transfers control back to Step **608**. Otherwise the error locations and values have been validated and Step **620** records them and transfers control to FIG. **17** for error correction.

The time required for executing looping control and maintaining counters in FIG. **6** may be eliminated by replacing FIG. **6** with the repeated sequential blocks of FIG. **6A**. Steps **650** through **65**x sequentially test coefficients R_i for i=I to I+t−**3**. If any of these tests fails, the locations and values are invalid and control is transferred to FIG. **8**. Otherwise the locations and values bave been validated and Step **660** records them and transfers control to FIG. **17** for error correction.

Referring to FIG. **7**, if either R_t or R_{t+1} is equal to zero, Step **700** transfers control to FIG. **8**. Otherwise Step **702** calculates the location L from equation (**13**). If L is invalid, Step **704** transfers control to FIG. **8**. Otherwise Step **706** calculates the error value $E=R_t/f(t,L)$ and prepares to validate L and E by initializing counters i=t, j=t−1, and k=2. Step **710** decrements counter i. If $R_1 \neq E*f(i,L)$, Step **712** transfers control to Step **720**. Otherwise Step **714** decrements counter j. If counter j is then still greater than zero, step **716** transfers control back to Step **710**. Otherwise L and E have been validated and Step **718** records them and transfers control to FIG. **17** for error correction. When a coefficient R_i fails the test of Step **712**, Step **720** decrements counter k. If counter k is then still greater than zero, Step **722** transfers control back to Step **710**. Otherwise L and E are invalid and control is transferred to FIG. **8**.

Referring to FIG. **8**, Step **800** initializes all $S_i=R_0$ and initializes counters j=1 and JXM=m_0. The purpose of counter JXM is to maintain the quantity j*m_0 for successive values of j by repaeated additions of m_0 and not by multiplication. If $R_j=0$, Step **810** transfers control to Step **850** since R_j does not alter the coefficients S_i. Otherwise Step **820** computes $k=\text{LOG }[R_j]+JXM$ and adds α^k to S_0. Step **822** computes $k=k+j$ and adds α^k to S_1. Steps **824** to **8xx** repeat the operations of Step **822** to compute and add the contribution of R_j to S_i for i=2 to MIN(2t−1,11). Step **850** increments counter j and computes $JXM=JXM+m_0$. If counter j is then still less than d−1, Step **860** transfers control back to Step **810**. Otherwise the coefficients S_i for i=0 to MIN(2t−1,11) have been calculated and control is transferred to FIG. **9**.

Referring to FIG. **9**, Step **900** initializes the polynomials, parameters, and counters. Step **910** computes the nth discrepancy value d_n. If d_n is equal to zero, Step **950** transfers control to Step **960**. Otherwise Step **952** updates $\sigma(x)$. If $1_n \geqq 1_k+n-k$, Step **954** transfers control to Step **958**. Otherwise Step **956** updates $\sigma_k(x)$ and other parameters. Step **958** updates $\sigma_p(x)$. Step **960** increments counter n. If $n<t+1_n$, Step **970** transfers control to Step **972**. If $n\neq 12$, Step **972** transfers control back to Step **910** to perform another iteration; otherwise FIG. **10** computes the remaining coefficients S_i before transferring control back to Step **910**. Otherwise if $\mathbf{1}_n$, the degree of $\sigma(x)$, is greater than t, the number of errors the code can correct, Step **980** exits the correction procedure unsuccessfully. If $1_n=t$, Step **980** transfers control to Step **982**; if n=d−2, one additional iteration is required before terminating the algorithm so Step

982 transfers control back to Step **910**. Otherwise we are assured that we have generated a valid error locator polynomial and control is transferred to FIG. **11**.

The evaluation of the nth discrepancy d_n of step **920** may be implemented with repeated sequential blocks as in FIG. **9**A. Step **920** initializes $d_n=0$. Step **930** uses $\mathbf{1}_n$ as an index into a table of software jump addresses to transfer control to the appopriate starting point in Steps **940** to **94**x for the evaluation of d_n. Multiplying S_n by σ_{p0} is not required in Step **94**x since σ_{p0} is always equal to one. When d_n has been computed, control is transferred to Step **950** of FIG. **9**.

Referring to FIG. **10**, operation is similar to that of FIG. **8**. After the remaining coefficients of S(x) have been calculated, control is transferred back to Step **910** of FIG. **9**.

Referring to FIG. **11**, Step **1100** initializes counters $i=0$, $j=\mathbf{1}_n=$the degree of $\sigma(x)$, and $k=0=$the number of error locations which have been found. If j is less than five, Step **1110** transfers control to FIG. **13**. Otherwise Step **1130** evaluates $\alpha-^{ji}*\sigma(x)$ at $x=\alpha^i$. If the result A is equal to zero, a root of $\sigma(x)$ has been found and Step **1160** transfers control to Step **1170**, which forms $L=-i$ before transferring control to FIG. **12**. Otherwise Step **1180** decrements counter i in the finite field. If counter i is then not equal to zero, Step **1190** transfers control back to Step **1130** to evaluate $\sigma(x)$ at the next location. Otherwise all possible locations have been tested without locating all the errors; therefore the correction correction procedure is exited unsuccessfully.

FIG. **11**A shows modifications to FIG. **11** necessary for efficient evaluation of $\alpha-^{ji}*\sigma(x)$ using repeated sequential blocks. Step **1120** is added to use counter j to compute an initial software jump address PTR=ADR(-Step **1140**)+$(t-j)*K$ where K is the size of each Step **1140** through **114**x. Step **1130** initializes A=0, then Step **1132** uses PTR to transfer control into Steps **1140** to **114**x, which sequentially evaluate $\alpha-^{ji}*\sigma(x)$ at $x=\alpha^i$ using Horner's rule. Step **1170** is modified to update PTR=PTR+K to reflect the decrementing of j performed in FIG. **12**.

Referring to FIG. **12**, Step **1200** increments counter k, records L, decrements counter j, then initializes D=1 and $N=S_j$. Step **1202** divides $\sigma(x)$ by $(x\oplus\alpha^L)$ and calculates the numerator and denominator of $E'=E*\alpha^{L*m}0$. If the new σ_j is equal to zero, the new $\sigma(x)$ has a root equal to zero, which is not the finite field antilogarithm of any error location; Step **1204** exits the correction procedure unsuccessfully. If the numerator is equal to zero, the computed error value is equal to zero, which is not a valid error value, so Step **1204** exits the correction procedure unsuccessfully. If the denominator is equal to zero, the error value cannot be computed, since division by zero in a finite field is undefined; Step **1204** exits the correction procedure unsuccessfully. If σ_j, the numerator, and the denominator are each not eaqual to zero, Step **1204** transfers control to Step **1206**, which calculates and records $E=\alpha^{-Lm}0*N/D$. If counter j is equal to zero, Step **1208** transfers control to FIG. **17** for error correction. Otherwise Step **1210** adjusts the coefficients of S(x) to remove the effects of the error just found and transfers control to FIG. **13**.

Referring to FIG. **13**, if more than four errors remain, Step **1300** transfers control to Step **1180** of FIG. **11** to search for another error location by evaluating $\sigma(x)$. If four errors remain, Step **1300** calls the quartic solution subroutine of FIG. **16**. If three errors remain, Step **1302** transfers control to Step **1304**, which sets parameters for and calls the cubic solution subroutine of FIG. **15**. If two errors remain, Step **1302** transfers control to Step **1306**, which sets parameters for and calls the quadratic solution subroutine of FIG. **14**. Otherwise one error remains and Step **1302** transfers control to Step **1308**.

If σ_1 is equal to zero, Step **1308** exits the correction procedure unsuccessfully, since the finite field logarithm of zero is undefined. Otherwise Step **1310** determines L=LOG $[\sigma_1]$ and transfers control to FIG. **12**.

Likewise, if one of the subroutines successfully determines an error location, Step **1320** transfers control to FIG. **12**. Otherwise, the correction procedure is exited unsuccessfully.

On entry to FIG. **14**, the parameters c_1 and c_2 describe the quadratic equation

$$x^2\oplus c_1*x\oplus c_2=0.$$

If $c_1=0$, the equation has a repeated root. If $c_2=0$, one of the roots is zero, whose log is undefined. If $c_1=0$ or $c_2=0$, Step **1400** exits the subroutine unsuccessfully. Otherwise Step **1402** determines a transformed root Y_1. If Y_1 is invalid, Step **1404** exits the subroutine unsuccessfully. Otherwise Step **1406** calculates the root X and its log L and returns successfully.

On entry to FIG. **15**, the parameters c_1, c_2, and c_3 describe the cubic equation

$$x^3\oplus c_1*x^2\oplus c_2*x\oplus c_3=0.$$

Step **1500** calculates the transform parameters A and B. If B is equal to zero, Step **1502** exits the subroutine unsuccessfully. Otherwise Step **1504** determines a root V of the quadratic equation

$$v^2 \oplus *v \oplus \frac{A^3}{B^2} = 0.$$

using the QUAD table. If no such root exists, Step **1504** produces zero and Step **1506** exits the subroutine unsuccessfully. Otherwise Step **1508** computes U. If U is not the cube of some finite field value T, Step **1510** exits the subroutine unsuccessfully. Otherwise Step **1512** calculates T and a root X of the cubic equation. If X is equal to zero, Step **1514** exits the subroutine unsuccessfully. Otherwise Step **1516** calculates the log L of the root X and returns successfully.

On entry to FIG. **16**, the parameters σ_1, σ_2, σ_3, and σ_4 describe the quartic equation

$$x^4\oplus\sigma_1*x^3\oplus\sigma_2*x^2\oplus\sigma_3*x\oplus\sigma_4=0.$$

If σ_1 is equal to zero, Step **1600** transfers contol to Step **1610**; if σ_3 is equal to zero, the quartic equation has repeated roots, so Step **1610** exits the subroutine unsuccessfully. Otherwise Step **1612** assigns $b_i=\sigma_i$ for i=2 to 4 and transfers control to Step **1620**. If σ_1 is not equal to zero, Step **1600** transfers control to Step **1602**, which calculates the numerator and denominator of transform parameter b_4. If the denominator of b_4 is equal to zero, Step **1604** exits the subroutine unsuccessfully. Otherwise Step **1606** calculates the transform parameters b_4, b_3, and b_2 and transfers control to Step **1620**.

Step **1620** sets parameters for and calls the cubic solution subroutine of FIG. **15**. If this returns unsuccessfully, Step **1622** exits the subroutine unsuccessfully.

Otherwise Step **1630** assigns Q=X and sets parameters for and calls the quadratic solution subroutine of FIG. **14.** If this returns unsuccessfully, Step **1632** exits the subroutine unsuccessfully. Otherwise Step **1640** sets parameters for and calls the quadratic solution subroutine of FIG. **14.** If this returns unsuccessfully, Step **1642** exits the subroutine unsuccessfully. Otherwise if σ_1 is equal to zero, Step **1650** returns L successfully. Otherwise Step **1660** computes X. If X is equal to zero, Step **1662** exits the subroutine unsuccessfully. Otherwise Step **1670** computes and returns L successfully.

Referring to FIG. **17,** on entry counter k is equal to the number of errors found, arrays L(*) and E(*) hold the pairs of error locations and values, D is the number of symbols in the data buffer and parameter DMN holds the reverse displacement plus one from the last symbol in the data buffer to the last data symbol in the current interleave. Step **1700** calculates the forward displacement from the first symbol in the data buffer to the last data symbol of the current interleave. Step **1710** fetches the location L=L(k) of the next error to be corrected. If L is less than d−1, the symbol in error is a check symbol and need not be corrected, so Step **1720** transfers control to **1760.** Otherwise Step **1730** calculates a forward displacement F within the data buffer from the reverse displacement L within the codeword interleave based on the data buffer length and number of interleaved codewords. If F is less than zero and therefore invalid, Step **1740** exits the correction procedure unsuccessfully. Otherwise Step **1750** corrects the data symbol in error with the error value E(k). Step **1760** decrements counter k. If counter k is then still greater than zero, Step **1770** transfers control back to Step **1710** to correct additional errors. Otherwise all errors have been corrected and the correction procedure is exited successfully.

While the invention herein disclosed has been described by means of specific embodiments and applications thereof, numerous modifications and variations could be made thereto by those skilled in the art without departing from the spirit and scope of the present invention. It is therefore to be understood that within the scope of the appended claims, the invention may be practised otherwise than as specifically described herein.

What is claimed is:

1. A decoder for an error detection and correction system using a Reed-Solomon code or related code of degree d−1 for detection and correction of a plurality of errors in code words of n symbols comprised of k data symbols and d−1 check symbols, wherein each symbol is comprised of m binary bits of information and d, k, m, and n are positive integers, and further wherein $t=\mathrm{INT}((d-1)/2)\geqq 3$, said decoder comprising:

data buffer means for storing said k data symbols;

remainder generator means for dividing a codeword polynomial C(x) by a generator polynomial G(x) of said code and producing a remainder polynomial R(x) having remainder coefficients R_1;

remainder buffer means for storing said remainder coefficients R_i produced by said remainder generator means;

first table means comprising a table f(i,L) wherein each element is comprised of error correction information, and wherein $0\leqq i<d-1$, and $d-1<L<2^m-1$; and

processor means comprising

means for accessing said remainder buffer to retrieve said remainder,

means for applying said remainder to index said first table means and retrieve said correction information, and

means for applying said correction information to said data symbols in said data buffer to correct symbols that are in error.

2. The decoder of claim **1** wherein each element of said first table means is comprised of a coefficient of the x^i term of x^L MOD G(x).

3. The decoder of claim **1** wherein each element of said first table means is comprised of a finite field logarithm of a coefficient of the x^i term of x^L MOD G(x).

4. The decoder of claim **1** further comprising a second table means comprising a table $L_{a,b}$ wherein each non-zero element is defined as

$$L_{a,b}\left[\frac{f(b,L)}{f(a,L)}\right]=L$$

wherein f(i,L) is a coefficient of the x^i term of x^L MOD G(x), L varies from d−1 to 2^m-2 and further wherein $0<a<d-1$, $0<b<d-1$, and $a\neq b$.

5. The decoder of claim **1** further comprising a third table means comprising a table $L_{a,b}$ wherein each non-zero element is defined as

$$L_{a,b}\left[\mathrm{LOG}\left[\frac{f(b,L)}{f(a,L)}\right]\right]=L$$

wherein f(i,L) is a coefficient of the x^i term of x^L MOD G(x), LOG [f(b,L)/f(a,L)] is the finite field logarithm of f(b,L)/f(a,L), L varies from d−1 to 2^m-2 and further wherein $0<a<d-1$, $0\leqq b<d-1$, and $a\neq b$.

6. The decoder of claim **5** wherein said means for applying said remainder comprises:

means for determining a location L of a data symbol error;

means for calculating a value E of said data symbol error; and

means for validating said location L and said value E of said data symbol error.

7. The decoder of claim **6** wherein said means for determining said location L comprises:

means for calculating an index value

$$\mathrm{LOG}\left[\frac{R_b}{R_a}\right],$$

wherein R_a and R_b are two of said coefficients R_i;

means for applying said index value to said third table means to retrieve said location L of said data symbol error.

8. The decoder of claim **6** wherein said means for determining said location L comprises:

means for calculating L according to

$$L=\mathrm{LOG}\left[\frac{\alpha^a\oplus\alpha^b*\frac{G'_a}{G'_b}*\frac{R_b}{R_a}}{1\oplus\frac{G'_a}{G'_b}*\frac{R_b}{R_a}}\right]\tag{13}$$

wherein G_i' is a coefficient of the x^i term of

$$G'(x) = \overset{d-3}{\underset{i=0}{\pi}} (x \oplus \alpha^{m0+i}).$$

9. The decoder of claim **6** wherein said means for calculating said value E comprises:

means for applying said location L to produce an index value for element f(a,L) of said first table means;

means for applying said index value to said first table means to retrieve said element f(a,L); and

means for dividing a coefficient R_a by said element f(a,L) to produce said value E of said data symbol error.

10. The decoder of claim **6** wherein said means for validating said location L and said value E comprises

means for calculating a finite field logarithm of said value E;

means for testing a plurality t of said remainder coefficients R_i wherein i≠a and i≠b, each test comprising a sequential repeated block comprising:

means for adding said finite field logarithm of said error value E to an element f(i,L) of said first table means to produce a finite field logarithm of a test value;

means for calculating a finite field antilogarithm of said finite logarithm of said test value; and

means for comparing said finite field antilogarithm of said test value to said coefficient R_i;

means for counting a number of said tests wherein said finite finite field antilogarithm of said test value is not equal to said remainder coefficient R_i;

means for recording an indicium when said number is less than two; and

means responsive to said indicium for using said location L and value E to correct said data symbol error.

11. The decoder of claim **5** wherein said means for applying said remainder comprises:

means for computing parameters σ_1 and σ_2;

means for determining locations L_1 and L_2 of two data symbol errors from σ_1 and σ_2;

means for calculating values E_1 and E_2 of said two data symbol errors; and

means for validating said locations and said values of said two data symbol errors.

12. The decoder of claim **11** wherein said means for computing said parameters σ_1 and σ_2 comprises:

means computing non-zero parameters D, N_1, and N_2 according to

$$N_1 = A_{ab}R_aR_b \oplus A_{ac}R_aR_c \oplus A_{ad}R_aR_d \oplus A_{bc}R_bR_c \oplus A_{bd}R_bR_d \oplus A_{cd}R_cR_d$$

$$N_2 = B_{ab}R_aR_b \oplus B_{ac}R_aR_c \oplus B_{ad}R_aR_d \oplus B_{bc}R_bR_c \oplus B_{bd}R_bR_d \oplus B_{cd}R_cR_d$$

$$D = C_{ab}R_aR_b \oplus C_{ac}R_aR_c \oplus C_{ad}R_aR_d \oplus C_{bc}R_bR_c \oplus C_{bd}R_bR_d \oplus C_{cd}R_cR_d$$

wherein pre-computed constants A_{ab}, A_{ac}, A_{ad}, A_{bc}, A_{bd}, A_{cd}, B_{ab}, B_{ac}, B_{ad}, B_{bc}, B_{bd}, B_{cd}, C_{ab}, C_{ac}, C_{ad}, C_{bc}, C_{bd}, and C_{cd} are functions of a, b, c, and d given by:

$$A_{ab} = (\alpha^{2a} \oplus \alpha^{2b})^*(G''_{c-1}{}^*G''_d \oplus G''_c{}^*G''_{d-1}),$$

$$A_{ac} = (\alpha^{2a} \oplus \alpha^{2c})^*(G''_{b-1}{}^*G''_d \oplus G''_b{}^*G''_{d-1}),$$

$$A_{ad} = (\alpha^{2a} \oplus \alpha^{2d})^*(G''_{b-1}{}^*G''_c \oplus G''_b{}^*G''_{c-1}),$$

$$A_{bc} = (\alpha^{2b} \oplus \alpha^{2c})^*(G''_{a-1}{}^*G''_d \oplus G''_a{}^*G''_{d-1}),$$

$$A_{bd} = (\alpha^{2b} \oplus \alpha^{2d})^*(G''_{a-1}{}^*G''_c \oplus G''_a{}^*G''_{c-1}),$$

$$A_{cd} = (\alpha^{2c} \oplus \alpha^{2d})^*(G''_{a-1}{}^*G''_b \oplus G''_a{}^*G''_{b-1}),$$

$$B_{ab} = (\alpha^{a*}\alpha^{2b} \oplus \alpha^{b*}\alpha^{2a})^*(G''_{c-1}{}^*G''_d \oplus G''_c{}^*G''_{d-1}),$$

$$B_{ac} = (\alpha^{a*}\alpha^{2c} \oplus \alpha^{c*}\alpha^{2a})^*(G''_{b-1}{}^*G''_d \oplus G''_b{}^*G''_{d-1}),$$

$$B_{ad} = (\alpha^{a*}\alpha^{2d} \oplus \alpha^{d*}\alpha^{2a})^*(G''_{b-1}{}^*G''_c \oplus G''_b{}^*G''_{c-1}),$$

$$B_{bc} = (\alpha^{b*}\alpha^{2c} \oplus \alpha^{c*}\alpha^{2b})^*(G''_{a-1}{}^*G''_d \oplus G''_a{}^*G''_{d-1}),$$

$$B_{bd} = (\alpha^{b*}\alpha^{2d} \oplus \alpha^{d*}\alpha^{2b})^*(G''_{a-1}{}^*G''_c \oplus G''_a{}^*G''_{c-1}),$$

$$B_{cd} = (\alpha^{c*}\alpha^{2d} \oplus \alpha^{d*}\alpha^{2c})^*(G''_{a-1}{}^*G''_b \oplus G''_a{}^*G''_{b-1}),$$

$$C_{ab} = (\alpha^{a} \oplus \alpha^{b})^*(G''_{c-1}{}^*G''_d \oplus G''_c{}^*G''_{d-1}),$$

$$C_{ac} = (\alpha^{a} \oplus \alpha^{c})^*(G''_{b-1}{}^*G''_d \oplus G''_b{}^*G''_{d-1}),$$

$$C_{ad} = (\alpha^{a} \oplus \alpha^{d})^*(G''_{b-1}{}^*G''_c \oplus G''_b{}^*G''_{c-1}),$$

$$C_{bc} = (\alpha^{b} \oplus \alpha^{c})^*(G''_{a-1}{}^*G''_d \oplus G''_a{}^*G''_{d-1}),$$

$$C_{bd} = (\alpha^{b} \oplus \alpha^{d})^*(G''_{a-1}{}^*G''_c \oplus G''_a{}^*G''_{c-1}),$$

$$C_{cd} = (\alpha^{c} \oplus \alpha^{d})^*(G''_{a-1}{}^*G''_b \oplus G''_a{}^*G''_{b-1}),$$

and G_i'' is a coefficient of the x^i term of

$$G''(x) = \overset{d-4}{\underset{i=0}{\pi}} (x \oplus \alpha^{m0+i})$$

with G_{-1}'' defined as zero; and

means for computing $\sigma_1 = N_1/D$ and $\sigma_2 = N_2/D$.

13. The decoder of claim **11** wherein t≧4 and selected coefficients R_a, R_b, R_c, and R_d of said coefficients R_i are each not equal to zero.

14. The decoder of claim **11** wherein said means for calculating said values E_1 and E_2 of said two data symbol errors comprises:

means for computing non-zero parameters D, N_1, and N_2 according to

$$D = f(a,L_1)^*f(b,L_2) \oplus f(b,L_1)^*f(a,L_2);$$

$$N_1 = R_a{}^*f(b,L_2) \oplus R_b{}^*f(a,L_2),$$

$$N_2 = R_a{}^*f(b,L_1) \oplus R_b{}^*f(a,L_1);\text{ and}$$

means for computing $E_1 = N_1/D$ and $E_2 = N_2/D$.

15. The decoder claim **11** wherein said means for validating said locations and said values comprises:

means for calculating finite field logarithms of said values E_1 and E_2;

means for testing a plurality t−2 of said remainder coefficients R_i wherein i≠a, i≠b, i≠c, and i≠d, each test comprising a sequential repeated block comprising:

means for adding said finite field logarithms of said values E_1 and E_2 to respective elements f(i,L_1) and f(i,L_2) of said first table means to produce finite field logarithms of test values,

means for calculating finite field antilogarithms of said finite field logarithms of said test values;

means for calculating an EXCLUSIVE-OR sum of said finite field antilogarithms of said test values; and

means for comparing said EXCLUSIVE-OR sum of said finite field antilogarithms of said test values to said coefficient R_i;

means for recording an indicium when each said EXCLUSIVE-OR sum of said finite field antilogarithms of said test values is equal to said coefficient R_i; and

means responsive to said indicium for using said locations and values to correct said data symbol errors.

16. The decoder of claim **1** wherein said means for applying said remainder comprises:

means for counting a number of non-zero coefficients R_i in a plurality of $t+1$ of said coefficients R_i in said remainder buffer and recording an indicium when said number is less than three; and

means responsive to said indicium for terminating error correction successfully.

17. The decoder of claim **1** wherein said means for applying said remainder comprises:

means for validating locations L_j and values E_j of e errors comprising means for testing a plurality of said remainder coefficients R_i according to the equations

$$R_i = \sum_{j=1}^{e} o\, E_j * f(i,L_j).$$

18. The decoder of claim **1** wherein said processor means comprises:

means for computing a syndrome polynomial S(x) from said remainder polynomial R(x);

means for generating an error location polynomial $\sigma(x)$ from said syndrome polynomial S(x);

means responsive to said error locator polynomial $\sigma(x)$ for locating errors; and

means responsive to said error locator polynomial $\sigma(x)$ and said syndrome polynomial S(x) for evaluating errors.

19. The decoder of claim **18** wherein said means for computing said syndrome polynomial comprises:

(1) means for initializing a coefficient S_0 and all other coefficients S_i of said syndrome polynomial S(x) to a coefficient R_0 of said remainder polynomial R(x);

(2) means for initializing a counter j to l;

(3) means for computing a finite field logarithmic partial result comprising a MODULO 2^m-1 sum of a finite field logarithm of a non-zero coefficient R_j and $j*m_0$;

(4) means for calculating a finite field antilogarithm of said partial result;

(5) means for EXCLUSIVE-OR adding said finite field antilogarithm of said partial result to said coefficient S_0;

(6) means for MODULO 2^m-1 adding said counter j to said partial result;

(7) means for calculating a finite field antilogarithm of said partial result;

(9) means for EXCLUSIVE-OR adding said finite field antilogarithm of said MODULO 2^m-1 sum and one of said coefficients S_i;

(10) means for repeating said means (6) through (9) for said coefficients S_i wherein $i=2$ to MIN(d−2,11);

(11) means for incrementing said counter j; and

(12) means for repeating said means (3) through (11) for said coefficients R_j wherein $j=2$ to $d-2$.

20. The decoder of claim **18** wherein said means for generating said error locator polynomial comprises:

means for computing an nth discrepancy d_n comprising sequential repeated blocks, each said block comprising means for calculating a MODULO 2^m-1 sum of said nth discrepancy d_n and a finite field product of a coefficient σ_{pi} and a coefficient S_{n-i};

means using a degree l_n of the nth error locator polynomial as an index into a table of software addresses of each said block;

means for recording an indicium when said parameter n is equal to twelve; and

means responsive to said indicium for generating coefficients S_i for $i=11$ to $d-2$.

21. The decoder of claim **18** wherein said means for locating said errors comprises:

means for recording an indicium when said error locator polynomial $\sigma(x)$ is of degree j greater than four;

means responsive to said indicium for locating one of said errors comprising evaluating said error locator polynomial $\sigma(x)$ for successive values of L until

$$A = \alpha^{jL} * \sigma(x)|_{\alpha - L} = 0,$$

said means for evaluating comprising a sequence of repeated blocks each said block comprising means for calculating a finite field product of α^{-L} and an EXCLUSIVE-OR sum of said parameter A and a coefficient σ_i of said error locator polynomial $\sigma(x)$; and

means for maintaining a software address of a starting point for next said evaluation.

22. The decoder of claim **18** wherein said means for locating said errors comprises:

means for recording an indicium when said error locator polynomial $\sigma(x)$ is of degree j less than or equal to four; and

means responsive to said indicium for locating one of said errors comprising:

means for calculating a finite field logarithm of a root of a quartic equation in a finite field;

means for calculating a root and a finite field logarithm of said root of a cubic equation in a finite field;

means for calculating a root and a finite field logarithm of said root of a quadratic equation in a finite field; and

means for calculating a finite field logarithm of a root of a linear equation in a finite field.

23. The decoder of claim **18** wherein said means for evaluating said errors comprises:

means for dividing said error locator polynomial $\sigma(x)$ by $(x \oplus \alpha^L)$ to produce a new error locator polynomial $\sigma(x)$ and calculating an error value E from said syndrome polynomial S(x) and said new error locator polynomial $\sigma(x)$, comprising a single software loop comprising:

(1) means for initializing a counter $g=1$, a remainder $R=1$, a denominator $D=1$, and a numerator $N=\sigma_j$;

(2) means for calculating a MODULO 2^m-1 sum of said remainder R and a finite field product of

a finite field antilogarithm of said location L and said remainder R;

(3) means for storing said MODULO 2^m-1 sum as said remainder R and as a coefficient σ_g of said error locator polynomial $\sigma(x)$;

(4) means for calculating a MODULO 2^m-1 sum of said remainder R and a finite field product of a finite field antilogarithm of said location L and said denominator D;

(5) means for storing said MODULO 2^m-1 sum as said denominator D;

(6) means for calculating a MODULO 2^m-1 sum of said numerator N and a finite field product of said remainder R and a coefficient S_{j-g} of said syndrome polynomial S(x);

(7) means for storing said MODULO 2^m-1 sum as said numerator N;

(8) means for incrementing said counter g; and

(9) means for repeating said means (2) through (8) for values of said counter g up to and including j;

means for recording an indicium when R, D, or N is equal to zero after the operation of said means (1) through (9);

means responsive to said indicium for terminating error correction unsuccessfully;

means for calculating a finite field quotient of said numerator N and said denominator D;

means for recording a finite field logarithm of said finite field quotient as a parameter E';

means for calculating a finite field product of said finite field quotient and a finite field antilogarithm of $-L^*m_0$;

means for recording said finite field product as said error value E; and

means for adjusting coefficients of said syndrome polynomial S(x) comprising a software loop comprising:

(a) means for initializing a counter g=0;

(b) means for calculating a finite field antilogarithm of said parameter E';

(c) means for calculating a MODULO 2^m-1 sum of said finite field antilogarithm and a coefficient S_i of said syndrome polynomial S(x);

(d) means for storing said MODULO 2^m-1 sum as said coefficient S_i;

(e) means for calculating a MODULO 2^m-1 sum of said parameter E' and said location L;

(f) means for storing said MODULO 2^m-1 sum as said parameter E';

(g) means for incrementing said counter g; and

(h) means for repeating said means (b) through (g) for values of said counter g up to and including j.

24. The decoder of claim **1** wherein m=8, t=8, G(x) is a GF(256) polynomial

$$G(x) = \underset{i=0}{\overset{15}{\pi}} (x \oplus \alpha^i),$$

and α^i are elements of a finite field generated by a GF(2) polynomial

$$x^8 \oplus x^4 \oplus x^3 \oplus x^2 \oplus 1.$$

25. The decoder of claim **1** wherein m=8, t=8, G(x) is a GF(256) polynomial

$$G(x) = \underset{i=0}{\overset{15}{\pi}} (x \oplus \alpha^{m0+i}),$$

$m_0=120$, and α^i are given by

$$\alpha^i=(\text{beta}^i)^{88},$$

wherein beta^i are elements of a finite field generated by a GF(2) polynomial

$$x^8 \oplus x^5 \oplus x^3 \oplus x^2 \oplus 1.$$

26. The decoder of claim **1** wherein m=8, t=4, G(x) is a GF(256) polynomial

$$G(x) = \underset{i=0}{\overset{7}{\pi}} (x \oplus \alpha^{m0+i}),$$

$m_0=124$, and α^i are given by

$\alpha^i=(\text{beta}^i)^{88}$,

wherein beta^i are elements of a finite field generated by a GF(2) polynomial

$$x^8 \oplus x^5 \oplus x^3 \oplus x^2 \oplus 1.$$

27. The decoder of claim **1** wherein m=8, t=2, G(x) is a GF(256) polynomial

$$G(x) = \underset{i=0}{\overset{3}{\pi}} (x \oplus \alpha^{m0+i}),$$

$m_0=126$, and α^i are given by

$$\alpha^i=(\text{beta}^i)^{88},$$

wherein beta^i are elements of a finite field generated by a GF(2) polynomial

$$x^8 \oplus x^5 \oplus x^3 \oplus x^2 \oplus 1.$$

28. In a decoder for an error detection and correction system using a Reed-Solomon code or related code of degree d−1 for detection and correction of a plurality of errors in codewords of n symbols comprised of k data symbols and d−1 check symbols, wherein each symbol is comprised of m binary bits of information and d, k, m, and n are positive integers, and further wherein $t=\text{INT}((d-1)/2)\geqq 3$, an error decoding method comprising the steps of:

storing said k data symbols in a data buffer;

generating a remainder polynomial R(x) having remainder coefficients R_i by dividing a codeword polynomial C(x) by a generator polynomial G(x) of said code;

storing said remainder coefficients in a remainder buffer;

applying said remainder coefficients to index a first table f(i,L), each element of said table being comprised of a coefficient of the x^i term of x^L MOD G(x) wherein L varies from d−1 to 2^m-2 and i varies from 0 to d−2, to produce correction information;

applying said correction information to said data symbols in said data buffer to correct symbols that are in error.

29. The method of claim **28** wherein said step of applying said remainder coefficients further comprises the steps of:

counting a number of non-zero coefficients R_i in a plurality t+2 of said coefficients R_i in said remainder buffer; and

terminating error correction successfully when said number is less than three.

30. The method of claim **28** wherein said step of applying said remainder coefficients further comprises the steps of:

determining a location L of a data symbol error;

calculating a value E of said data symbol error; and

validating said location L and said value E of said data symbol error.

31. The method of claim **30** wherein said step of determining said location L comprises the steps of:

calculating an index value from two coefficients R_a and R_b of said coefficients R_i:

$$\mathrm{LOG}\left[\frac{R_b}{R_a}\right]; \text{ and}$$

applying said index value to a second table $L_{a,b}$ wherein each non-zero element is given by

$$L_{a,b}\left[\mathrm{LOG}\left[\frac{f(b,L)}{f(a,L)}\right]\right] = L$$

for $L = d-1$ to $2^m - 1$ to retrieve said location L of said data symbol error.

32. The method of claim **30** wherein said step of determining said location L comprises calculating L according to

$$L = \mathrm{LOG}\left[\frac{\alpha^a \oplus \alpha^b * \frac{G_a'}{G_b'} * \frac{R_b}{R_a}}{1 \oplus \frac{G_a'}{G_b'} * \frac{R_b}{R_a}}\right] \tag{13}$$

wherein G_i' is a coefficient of the x^i term of

$$G'(x) = \prod_{i=0}^{d-3} (x \oplus \alpha^{m0+i}).$$

33. The method of claim **30** wherein said step of calculating said value E comprises the steps of:

using said location L and a number a wherein $0 \leqq a < d-1$ to produce an index value for an element f(a,L) of said first table;

retrieving said element f(a,L) of said first table referenced by said index value;

dividing a coefficient R_a by said element f(a,L) to produce said value E of said data symbol error.

34. The method of claim **30** wherein said step of validating said location L and said value E comprises the steps of:

calculating a finite field logarithm of said value E;

testing a plurality t of said remainder coefficients R_i wherein i≠a and i≠b, each test comprising a sequential repeated block comprising the steps of:

adding said finite field logarithm of said error value E to an element f(i,L) of said first table to produce a finite field logarithm of a test value;

calculating a finite field antilogarithm of said finite field logarithm of said test value; and

comparing said finite field antilogarithm of said test value to said coefficient R_i;

counting a number of said tests wherein said finite finite field antilogarithm of said test value is not equal to said remainder coefficient R_i; and

correcting said data symbol error using said location L and said value E when said number is less than two.

35. The method of claim **28** wherein said step of applying said remainder coefficients further comprises the steps of:

computing parameters σ_1 and σ_2;

determining locations L_1 and L_2 of two data symbol errors from σ_1 and σ_2;

calculating values E_1 and E_2 of said two data symbol errors; and

validating said locations and said values of said two data symbol errors.

36. The method of claim **35** wherein said step of computing said parameters σ_1 and σ_2 comprises the steps of:

computing non-zero parameters D, N_1, and N_2 according to

$$N_1 = A_{ab}R_aR_b \oplus A_{ac}R_aR_c \oplus A_{ad}R_aR_d \oplus A_{bc}R_bR_c \oplus A_{bd}R_bR_d \oplus A_{cd}R_cR_d$$

$$N_2 = B_{ab}R_aR_b \oplus B_{ac}R_aR_c \oplus B_{ad}R_aR_d \oplus B_{bc}R_bR_c \oplus B_{bd}R_bR_d \oplus B_{cd}R_cR_d$$

$$D = C_{ab}R_aR_b \oplus C_{ac}R_aR_c \oplus C_{ad}R_aR_d \oplus C_{bc}R_bR_c \oplus C_{bd}R_bR_d \oplus C_{cd}R_cR_d$$

wherein pre-computed constants A_{ab}, A_{ac}, A_{ad}, A_{bc}, A_{bd}, A_{cd}, B_{ab}, B_{ac}, B_{ad}, B_{bc}, B_{bd}, B_{cd}, C_{ab}, C_{ac}, C_{ad}, C_{bc}, C_{bd}, and C_{cd} are functions of a, b, c, and d given by:

$$A_{ab} = (\alpha^{2a} \oplus \alpha^{2b}) * (G''_{c-1} * G''_d \oplus G''_c * G''_{d-1}),$$

$$A_{ac} = (\alpha^{2a} \oplus \alpha^{2c}) * (G''_{b-1} * G''_d \oplus G''_b * G''_{d-1}),$$

$$A_{ad} = (\alpha^{2a} \oplus \alpha^{2d}) * (G''_{b-1} * G''_c \oplus G''_b * G''_{c-1}),$$

$$A_{bc} = (\alpha^{2b} \oplus \alpha^{2c}) * (G''_{a-1} * G''_d \oplus G''_a * G''_{d-1}),$$

$$A_{bd} = (\alpha^{2b} \oplus \alpha^{2d}) * (G''_{a-1} * G''_c \oplus G''_a * G''_{c-1}),$$

$$A_{cd} = (\alpha^{2c} \oplus \alpha^{2d}) * (G''_{a-1} * G''_b \oplus G''_a * G''_{b-1}),$$

$$B_{ab} = (\alpha^a * \alpha^{2b} \oplus \alpha^b * \alpha^{2a}) * (G''_{c-1} * G''_d \oplus G''_c * G''_{d-1}),$$

$$B_{ac} = (\alpha^a * \alpha^{2c} \oplus \alpha^c * \alpha^{2a}) * (G''_{b-1} * G''_d \oplus G''_b * G''_{d-1}),$$

$$B_{ad} = (\alpha^a * \alpha^{2d} \oplus \alpha^d * \alpha^{2a}) * (G''_{b-1} * G''_c \oplus G''_b * G''_{c-1}),$$

$$B_{bc} = (\alpha^b * \alpha^{2c} \oplus \alpha^c * \alpha^{2b}) * (G''_{a-1} * G''_d \oplus G''_a * G''_{d-1}),$$

$$B_{bd} = (\alpha^b * \alpha^{2d} \oplus \alpha^d * \alpha^{2b}) * (G''_{a-1} * G''_c \oplus G''_a * G''_{c-1}),$$

$$B_{cd} = (\alpha^c * \alpha^{2d} \oplus \alpha^d * \alpha^{2c}) * (G''_{a-1} * G''_b \oplus G''_a * G''_{b-1}),$$

$$C_{ab} = (\alpha^a \oplus \alpha^b) * (G''_{c-1} * G''_d \oplus G''_c * G''_{d-1}),$$

-continued

$$C_{ac} = (\alpha^a \oplus \alpha^c) * (G''_{b-1} * G''_d \oplus G''_b * G''_{d-1}),$$

$$C_{ad} = (\alpha^a \oplus \alpha^d) * (G''_{b-1} * G''_c \oplus G''_b * G''_{c-1}),$$

$$C_{bc} = (\alpha^b \oplus \alpha^c) * (G''_{a-1} * G''_d \oplus G''_a * G''_{d-1}),$$

$$C_{bd} = (\alpha^b \oplus \alpha^d) * (G''_{a-1} * G''_c \oplus G''_a * G''_{c-1}),$$

$$C_{cd} = (\alpha^c \oplus \alpha^d) * (G''_{a-1} * G''_b \oplus G''_a * G''_{b-1}),$$

and G_i'' is a coefficient of the x^i term of

$$G''(x) = \prod_{i=0}^{d-4} (x \oplus \alpha^{m0+i})$$

with G_{-1}'' defined as zero; and computing $\sigma_1 = N_1/D$ and $\sigma_2 = N_2/D$.

37. The method of claim **35** wherein $t > 4$ and selected coefficients R_a, R_b, R_c, and R_d of said coefficients R_i are each not equal to zero.

38. The method of claim **35** wherein said step of calculating said values E_1 and E_2 of said two data symbol errors comprises the steps of:

computing non-zero parameters D, N_1, and N_2 according to

$$D = f(a,L_1)*f(b,L_2) \oplus f(b,L_1)*f(a,L_2);$$

$$N_1 \cdot = R_a*f(b,L_2) \oplus R_b*f(a,L_2),$$

$$N_2 = R_a*f(b,L_1) \oplus R_b*f(a,L_1),$$

wherein R_a, R_b, R_c, and R_d are selected coefficients of said coefficients R_i; and computing $E_1 = N_1/D$ and $E_2 = N_2/D$.

39. The method of claim **35** wherein said step of validating said locations and said values of said two data symbol errors comprises the steps of:

calculating finite field logarithms of said values E_1 and E_2;

testing a plurality $t-2$ of said remainder coefficients R_i wherein $i \neq a$, $i \neq b$, $i \neq c$, and $i \neq d$, and further wherein each test comprises a sequential repeated block comprising the steps of:

adding said finite field logarithms of said values E_1 and E_2 to respective elements $f(i,L_1)$ and $f(i,L_2)$ of said first table to produce finite field logarithms of test values,

calculating finite field antilogarithms of said finite field logarithms of said test values;

calculating an EXCLUSIVE-OR sum of said finite field antilogarithms of said test values;

comparing said EXCLUSIVE-OR sum of said finite field antilogarithms of said test values to said coefficient R_i;

correcting said two data symbol errors using said locations and values when each said EXCLUSIVE-OR sum of said finite field antilogarithms of said test values is equal to said coefficient R_i.

40. The method of claim **28** wherein said step of applying said remainder coefficients further comprises validating locations L_j and values E_j of a plurality e of symbol errors by testing a plurality of said remainder coefficients R_i according to the equations

$$R_i = \sum_{j=1}^{e} o\, E_j*f(i,L_j).$$

41. The method of claim **28** wherein said step of applying said remainder coefficients further comprises the steps of:

computing a syndrome polynomial S(x) from said remainder polynomial R(x);

generating an error locator polynomial σ(x) from said syndrome polynomial S(x);

locating errors using said error locator polynomial σ(x); and

evaluating errors using said error locator polynomial σ(x) and said syndrome polynomial S(x).

42. The method of claim **41** wherein said step of computing said syndrome polynomial comprises the steps of:

(1) initializing a coefficient S_0 and all other coefficients S_i of said syndrome polynomial S(x) to a coefficient R_0 of said remainder polynomial R(x);

(2) initializing a counter j to 1;

(3) computing a finite field logarithmic partial result comprising a MODULO 2^m-1 sum of a finite field logarithm of a non-zero coefficient R_j and $j*m_0$;

(4) calculating a finite field antilogarithm of said partial result;

(5) EXCLUSIVE-OR adding said finite field antilogarithm of said partial result to said coefficient S_0;

(6) MODULO 2^m-1 adding said counter j to said partial result;

(7) calculating a finite field antilogarithm of said partial result;

(9) EXCLUSIVE-OR adding said finite field antilogarithm of said MODULO 2^m-1 sum and one of said coefficients S_i;

(10) repeating said steps (6) through (9) for said coefficients S_i wherein $i=2$ to $\mathrm{MIN}(d-2,11)$;

(11) incrementing said counter j; and

(12) repeating said steps (3) through (11) for said coefficients R_j wherein $j=2$ to $d-2$.

43. The method of claim **41** wherein said step of generating said error locator polynomial comprises the steps of:

computing an nth discrepancy d_n using sequential repeated blocks, each said block comprising calculating a MODULO 2^m-1 sum of said nth discrepancy d_n and finite field product of a coefficient σ_{pi} and a coefficient S_{n-i};

using a degree 1_n of an nth error locator polynomial σ(x) as an index into a table of software addresses of each said block;

generating coefficients S_i for $i=11$ to $d-2$ when said parameter is equal to twelve.

44. The method of claim **41** wherein said step of locating said errors when said error locator polynomial σ(x) is of degree j greater than four comprises the steps of:

evaluating said error locator polynomial σ(x) for successive values of L until

$$A = \alpha^{jL}*\sigma(x)\,|_{\alpha-L} = 0,$$

said step of evaluating comprising a sequence of repeated blocks, each said block comprising calculating a finite field product of α^{-L} and an EXCLU-

SIVE-OR sum of said parameter A and a coefficient σ_i of said error locator polynomial $\sigma(x)$; and

maintaining a software address of a starting point for next said evaluation.

45. The method of claim **41** wherein said step of locating said errors when said error locator polynomial $\sigma(x)$ is of degree j less than or equal to four comprises locating one of said errors using one of the steps of:

when j is equal to four, calculating a finite field logarithm of a root of a quartic equation in a finite field;

when j is equal to three, calculating a root and a finite field logarithm of said root of a cubic equation in a finite field;

when j is equal to two, calculating a root and a finite field logarithm of said root of a quadratic equation in a finite field; or

when j is equal to one, calculating a finite field logarithm of a root of a linear equation in a finite field.

46. The method of claim **41** wherein said step of evaluating said errors comprises the steps of:

dividing said error locator polynomial $\sigma(x)$ by $(x\oplus\alpha^{L})$ to produce a new error locator polynomial S(x) and calculating an error value E from said syndrome polynomial S(x) and said new error locator polynomial $\sigma(x)$, all in a single software loop comprising steps of:

(1) initializing a counter $g=1$, a remainder $R=1$, a denominator $D=1$, and a numerator $N=\sigma_j$;

(2) calculating a MODULO 2^m-1 sum of said remainder R and a finite field product of a finite field antilogarithm of said location L and said remainder R;

(3) storing said MODULO 2^m-1 sum as said remainder R and as a coefficient σ_g of said error locator polynomial $\sigma(x)$;

(4) calculating a MODULO 2^m-1 sum of said remainder R and a finite field product of a finite field antilogarithm of said location L and said denominator D;

(5) storing said MODULO 2^m-1 sum as said denominator D;

(6) calculating a MODULO 2^m-1 sum of said numerator N and a finite field product of said remainder R and a coefficient $S_{j\text{-}g}$ of said syndrome polynomial S(x);

(7) storing said MODULO 2^m-1 sum as said numerator N;

(8) incrementing said counter g; and

(9) repeating said steps (2) through (8) for values of said counter g up to and including j;

terminating error correction unsuccessfully when R, D, or N is equal to zero after completion of said steps (1) through (9);

calculating a finite field quotient of said numerator N and said denominator D;

recording said a finite field logarithm of said finite field quotient as a parameter E′;

calculating a finite field product of said finite field quotient and a finite field antilogarithm of $-L*m_0$;

recording said finite field product as said error value E; and

adjusting coefficients of said syndrome polynomial S(x) using steps of:

(a) initializing counter $g=0$;

(b) calculating a finite field antilogarithm of said parameter E′;

(c) calculating a MODULO 2^m-1 sum of said finite field antilogarithm and a coefficient S_i of said syndrome polynomial S(x);

(d) storing said MODULO 2^m-1 sum as said coefficient S_i;

(e) calculating a MODULO 2^m-1 sum of said parameter E′ and said location L;

(f) storing said MODULO 2^m-1 sum as said parameter E′;

(g) incrementing said counter g; and

(h) repeating said steps (b) through (g) for values of said counter g up to and including j.

47. In a decoder for an error detection and correction system using a Reed-Solomon code or related code of degree $d-1$ for detection and correction of a plurality of errors wherein a message block is comprised of N interleaved codewords of said code wherein codeword i is comprised of $n_i-(d-1)$ data symbols and $d-1$ check symbols comprising a total of D data symbols stored in a data buffer means and $N*(d-1)$ check symbols stored in a remainder buffer means where d, i, N, and n_i are positive integers, and further wherein the first check symbol in said remainder buffer means is remainder coefficient R_{d-2} of codeword D MOD N and the last symbol in said remainder buffer means is remainder coefficient R_0 of codeword $(D-1)$ MOD N, with other coefficients interleaved between, a method for accessing said data buffer means and said remainder buffer means for detection and correction of said errors comprising the steps of:

(1) initializing a parameter DMN equal to said number of data symbols D MODULO said number of interleaves N and initializing a counter I to zero;

(2) if said parameter DMN is equal to zero, resetting said parameter DMN to said number of interleaves N;

(3) computing a forward displacement within said remainder buffer means of coefficient R_0 of codeword I by calculating $N*(d-1)-DMN$;

(4) computing forward displacements within said remainder buffer means of other coefficients R_i of said codeword I by repeated subtraction of said number of interleaves N from said forward displacement of said coefficient R_0 of said codeword I;

(5) determining location(s) L_i and value(s) E_i of error(s) in said codeword I;

(6) computing a forward displacement within said data buffer means of a last data symbol within said codeword I as $F_{max}=d-DMN$;

(7) computing a forward displacement within said data buffer means of an error at a location $L_i>d-1$ as $F_i=F_{max}-N*L_i$;

(8) correcting said error at said forward displacement F_i using error value E_i;

(9) repeating said steps (7) and (8) for all errors in said codeword I;

(10) decrementing said parameter DMN and incrementing said counter I; and

(11) repeating said steps (2) through (10) for values of said counter I up to and including $N-1$.

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